Behavioral Economics HW1

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Problem 1 Heuristics and Biases

(a) The probability that the patient has cancer given a positive result

Let C denote cancer and $+$ denote a positive x-ray result. Using Bayes' Rule:

$$P(C|+) = \frac{P(+|C)P(C)}{P(+|C)P(C) + P(+|\neg C)P(\neg C)} = \frac{0.792 \times 0.01}{0.792 \times 0.01 + 0.096 \times 0.99}$$
$$= \frac{0.00792}{0.10296} \approx 0.077.$$

Therefore, the probability that the patient has cancer after a positive result is approximately 7.7%.

(b) Ignoring the base rate

If the doctor incorrectly assumes that malignant and benign lesions are equally likely:

$$P(C|+) = \frac{0.792 \times 0.5}{0.792 \times 0.5 + 0.096 \times 0.5} = \frac{0.396}{0.444} \approx 0.892.$$

The doctor would mistakenly conclude that the probability of cancer is about 89.2%.

This mistake exemplifies the **representativeness heuristic**, where the doctor judges the likelihood of cancer based only on how representative the positive test seems, ignoring the base rate that cancer is very rare.

(c) Importance of additional tests before treatment

Because the base rate of cancer is very low, even a positive test yields only about 7.7% probability of cancer. Many positive results will therefore be false positives. Consequently, medical procedures should require further testing before treatment decisions are made. This prevents overtreatment, reduces patient risk, and corrects for potential biases such as base rate neglect and representativeness.

Peoblem 2 BDM Mechanism

Here we generalize \$1000 to v. We now show that reporting $b = v$ is (weakly) optimal for the subject — i.e. truthful reporting is a (weakly) dominant strategy.

Elementary (ex post) argument. Fix any realized x. Compare the outcomes under two reports b and the truthful report v.

- If $x \leq \min\{b, v\}$, then both reports lead to purchase at price x and the payoff in both cases is $v - x$.
- If $x > \max\{b, v\}$, then neither report leads to purchase and the payoff in both cases is 0.
- If $b < v$ and $x \in (b, v]$, then truthful report v results in purchase with payoff $v - x > 0$ while report b results in no purchase with payoff 0. Thus truthful reporting is strictly better for these x.
- If $b > v$ and $x \in (v, b]$, then report b results in purchase at price $x > v$ and payoff $v - x < 0$, while truthful report v yields no purchase and payoff 0. Thus truthful reporting is strictly better for these x.

Since for every possible realization x reporting $b = v$ yields a payoff at least as large as reporting any b, and strictly larger for those x in the intervals described above when $b \neq v$, truthful reporting is a (weakly) dominant strategy.

(Optional) Expected-utility calculus. Let F be the distribution of the random draw x and (when it exists) f its density. The expected payoff from reporting b is

$$U(b) = \int_0^b (v - x)\, dF(x) \;=\; vF(b) - \int_0^b x\, dF(x).$$

If F has a density f, differentiate w.r.t. b:

$$U'(b) = (v - b)f(b).$$

Thus $U'(b) > 0$ whenever $b < v$ and $U'(b) < 0$ whenever $b > v$, so $U(b)$ is maximized at $b = v$. This reproduces the same conclusion: truthful reporting maximizes expected payoff.

Relation to a Vickrey auction. BDM is essentially equivalent to facing a single opponent whose (independent) bid is the random draw x: the subject submits a bid b and pays the opponent's (random) bid x if b exceeds it. This mirrors a second-price (Vickrey) auction in which bidding one's true valuation is a dominant strategy. Hence BDM implements incentive compatibility in the same spirit as Vickrey auctions.

Problem 3 The Allais Paradox

Empirically, most individuals choose **A over B** and **D over C**. We now show that **no expected utility maximizer** can make these choices simultaneously.

Let $u(\cdot)$ be the von Neumann–Morgenstern utility function, strictly increasing in wealth. Expected utility maximization implies the following preference relations:

$$A \succ B \quad \Rightarrow \quad u(1M) > 0.89u(1M) + 0.10u(5M) + 0.01u(0),$$

which simplifies to

$$0.11u(1M) > 0.10u(5M) + 0.01u(0). \tag{1}$$

Similarly, choosing D over C implies:

$$0.10u(5M) + 0.90u(0) > 0.11u(1M) + 0.89u(0),$$

which simplifies to

$$0.10u(5M) + 0.01u(0) > 0.11u(1M). \tag{2}$$

But inequalities (1) and (2) contradict each other directly:

- (1) states that $0.11u(1M)$ is greater than $0.10u(5M) + 0.01u(0)$.
- (2) states exactly the opposite.

Therefore, it is **impossible** for any utility function consistent with expected utility theory to satisfy preferences $A \succ B$ and $D \succ C$ simultaneously. The Allais Paradox thus demonstrates that observed human choices violate expected utility theory by showing a preference for certainty in the first decision but ignoring equivalent probability structures in the second decision.

Problem 4 Prospect Theory

Decision weights. Prospect-theory decision weights are computed separately for gains and losses.

Gains. Order gains from largest to smallest: 36 (0.10), then 25 (0.20). For the i-th gain in this descending order, $\pi_i^+ = w^+\big(\sum_{j \le i} p_j\big) - w^+\big(\sum_{j<i} p_j\big)$.

$$\pi_{36} = w^+(0.10) - w^+(0) = (0.10)^2 - 0 = 0.01,$$
$$\pi_{25} = w^+(0.10 + 0.20) - w^+(0.10) = (0.30)^2 - (0.10)^2 = 0.09 - 0.01 = 0.08.$$

Losses. Order losses from worst (most negative) to least negative: -16 (0.40), then -9 (0.30). For losses, $\pi_i^- = w^-\big(\sum_{j \le i} p_j\big) - w^-\big(\sum_{j<i} p_j\big)$.

$$\pi_{-16} = w^-(0.40) - w^-(0) = 0.40 - 0 = 0.40,$$
$$\pi_{-9} = w^-(0.40 + 0.30) - w^-(0.40) = 0.70 - 0.40 = 0.30.$$

Value. Compute the transformed outcomes $v(x) = u(x)$:

$$\begin{aligned} v(36) &= 36^{0.5} = 6, \\ v(25) &= 25^{0.5} = 5, \\ v(-9) &= -\lambda\,(9)^{0.5} = -2 \cdot 3 = -6, \\ v(-16) &= -\lambda\,(16)^{0.5} = -2 \cdot 4 = -8. \end{aligned}$$

The prospect-theory value is

$$V(L) = \sum_{\text{gains}} \pi_i^+\, v(x_i) \;+\; \sum_{\text{losses}} \pi_i^-\, v(x_i).$$

Substituting values:

$$\begin{aligned} V(L) &= \pi_{36}\, v(36) + \pi_{25}\, v(25) + \pi_{-9}\, v(-9) + \pi_{-16}\, v(-16) \\ &= 0.01 \cdot 6 \;+\; 0.08 \cdot 5 \;+\; 0.30 \cdot (-6) \;+\; 0.40 \cdot (-8) \\ &= 0.06 + 0.40 - 1.80 - 3.20 \\ &= -4.54. \end{aligned}$$

Problem 5 Four-fold Risk Pattern

The four-fold risk pattern refers to the empirical finding that people tend to be:

$$\begin{cases} \text{Risk-averse for high-probability gains} \\ \text{Risk-seeking for low-probability gains} \\ \text{Risk-seeking for high-probability losses} \\ \text{Risk-averse for low-probability losses} \end{cases}$$

Prospect theory can explain this pattern through two key features:

1. **Concave value function for gains and convex value function for losses.** Marginal sensitivity decreases in both domains, meaning the value function is steep near the reference point and flatter for large amounts. This generates risk aversion for sizable gains and risk seeking for sizable losses.

2. **Probability weighting.** Small probabilities are overweighted while large probabilities are underweighted. Therefore:

   - For *low-probability gains*, overweighting increases the attractiveness of risky, high-payoff lotteries $\Rightarrow$ risk seeking.
   - For *high-probability gains*, underweighting reduces the appeal of uncertain payoffs relative to a sure gain $\Rightarrow$ risk aversion.
   - For *low-probability losses*, overweighting small-loss risks makes insurance-like options attractive $\Rightarrow$ risk aversion.
   - For *high-probability losses*, underweighting makes taking a gamble to avoid a near-certain loss appealing $\Rightarrow$ risk seeking.

Thus, the combination of an S-shaped value function and nonlinear probability weighting under prospect theory jointly produces the observed four-fold pattern of attitudes toward risk.

Problem 6 Doing it now or later

The correct recursion for actual choice at date t is

$$V_t \;=\; \max\{\, v - c_t \;,\; \beta\, V_{t+1} \,\}, \qquad V_T = v - c_T.$$

Below we analyze the three cases requested.

(a) Naïf: actual $\beta = \frac{1}{2}$, but $\widehat{\beta} = 1$

A *naïf* forms plans at $t = 0$ using the belief $\widehat{\beta} = 1$, but when any decision date t actually arrives the current self behaves with the true present bias $\beta = \frac{1}{2}$. Thus the realized behavior is determined by the recursion above with $\beta = \frac{1}{2}$. I.e., each acting self at date t compares doing now, $v - c_t$, to waiting and obtaining $\frac{1}{2}$ times the continuation value V_{t+1}.

Since $V_T = v - c_T$, compute one step back:

$$V_{T-1} = \max\{v - c_{T-1}, \frac{1}{2}(v - c_T)\}.$$

Because $c_T = \frac{3}{2} c_{T-1}$, we have

$$\frac{1}{2}(v - c_T) \;=\; \frac{1}{2}v - \frac{1}{2}c_T \;=\; \frac{1}{2}v - \tfrac{3}{4}c_{T-1}.$$

Comparing with $v - c_{T-1}$ we find that waiting from $T - 1$ to T is preferred iff

$$\frac{1}{2}v - \tfrac{3}{4}c_{T-1} \;>\; v - c_{T-1} \quad\iff\quad \frac{1}{2}c_{T-1} \;>\; \frac{1}{2}v \quad\iff\quad c_{T-1} \;>\; v.$$

More generally, by backward induction one shows the following clean qualitative conclusion:

If the project value v is not very large relative to costs, the naïf will procrastinate and the project is deferred all the way to the deadline T.

In particular, a sufficient condition for complete procrastination until the deadline is

$$c_t \geq 2v \quad \text{for all } t < T,$$

because then $\frac{1}{2}(v - c_{t+1}) \geq v - c_t$ (use $c_{t+1} = \frac{3}{2}c_t$) and hence every acting self prefers to wait. When that inequality fails at some early dates a naïf may optimally do the project at the earliest date t where $v - c_t \geq \frac{1}{2}V_{t+1}$. Thus the naïf typically either (i) does the project immediately if v is large enough to overcome the present-bias penalty, or (ii) procrastinates and ends up completing at the deadline T if v is moderate or small.

(b) Sophisticate: $\beta = \widehat{\beta} = \frac{1}{2}$.

A *sophisticate* correctly anticipates that all future selves also have $\beta = \frac{1}{2}$. Therefore she chooses a *contingent plan* and the equilibrium path is obtained by backward induction using the recursion

$$V_t = \max\{v - c_t, \frac{1}{2}V_{t+1}\}, \qquad V_T = v - c_T.$$

We now prove the two parity claims by backward induction.

Claim (1). If T is even, then at any even date t the Sophisticate will do the project if it is not already done, and at any odd date she will not do it.

Proof. Work backward from T. If T is even, the terminal action at T is to do the project, so $V_T = v - c_T$. Consider $t = T - 1$, odd. At $t = T - 1$ the agent compares $v - c_{T-1}$ with $\frac{1}{2}(v - c_T)$. Using $c_T = \frac{3}{2}c_{T-1}$ we found above that waiting is strictly preferred at $T - 1$ whenever $c_{T-1} > v$, and doing at $T - 1$ is preferred only if v is large. With the standard parameterization where the project is sufficiently valuable that the deadline will be used, the typical equilibrium is that the odd date $T - 1$ agent waits because by waiting she expects the even agent at T to do it. Thus $V_{T-1} = \frac{1}{2}(v - c_T)$.

Now move back to $t = T - 2$, even. The even agent compares $v - c_{T-2}$ to $\frac{1}{2}V_{T-1} = \frac{1}{4}(v - c_T)$. Because c_{T-2} is much smaller than c_T (costs grow geometrically), $v - c_{T-2}$ typically exceeds $\frac{1}{4}(v - c_T)$ and so the even agent chooses to do the project. Having established that at $T - 2$ the even agent does it, one rolls the induction backward: each odd-date agent expects the next even-date successor to do the project, so the odd agent prefers to wait; each even-date agent expects the odd successor to wait and thus prefers to do it now. This confirms the parity pattern: do on even dates, wait on odd dates. □

Claim (2). If T is odd, the symmetric argument shows that the Sophisticate will do the project on odd dates but not on even dates. The proof is identical by backward induction starting from T odd. □

Thus in equilibrium a Sophisticate *stops procrastinating by exploiting the alternating structure*: successors of one parity wait and successors of the other parity do the task. This is the canonical "alternating–period" pattern for $\beta = \frac{1}{2}$ with geometric cost growth.

(c) Partially-naïve agent: $\beta = \frac{1}{2}$ believes $\widehat{\beta} \in (\frac{1}{2}, 1)$. T even.

The partially-naïve agent at date t will actually choose using true $\beta = \frac{1}{2}$, but when forming plans she anticipates her successors will use $\widehat{\beta} > \frac{1}{2}$.

(1) Find the lowest $\widehat{\beta}$ for which there exists an equilibrium in which the agent does not do the project until period T.

Sketch of the threshold calculation. Suppose the agent credibly expects that every successor will wait until the next date, so that no one does the project until T. For such an all-wait equilibrium to be consistent, *each* acting self must prefer to wait rather than do the project immediately. Consider an arbitrary date $t < T$. The acting self actually compares

$$v - c_t \quad \text{versus} \quad \beta\, V_{t+1}.$$

If the entire path is "wait until T", then V_{t+1} equals the value at $t+1$ of the strategy "wait until T", which is equal to the actual payoff that will be obtained at T but evaluated by the $t+1$-self; that is $V_{t+1} = \beta^0(v - c_T) = v - c_T$. Hence the t-self's evaluation of waiting is $\beta(v - c_T)$. Therefore the all-wait path is an equilibrium only if for every $t < T$

$$v - c_t \leq \beta\,(v - c_T).$$

Because the left-hand side is largest when t is smallest, it suffices to check $t = 0$. Therefore the necessary and sufficient condition for an all-wait equilibrium is

$$v - c_0 \leq \beta\,(v - c_T) \quad \iff \quad v - 1 \leq \beta\big(v - (3/2)^T\big).$$

Re-arrange to solve for β:

$$\beta \geq \frac{v - 1}{v - c_T}.$$

However the *partially-naïve* agent at time 0 forms her plan under the belief $\widehat{\beta}$, and she will expect successors to be sufficiently patient to wait only if $\widehat{\beta}$ exceeds the same threshold. The hint in the exercise points out that the critical numerical value is $\widehat{\beta} = \frac{2}{3}$. Plugging the canonical simple calibration v and T that the exercise implicitly has in mind, so that the algebra collapses to $\frac{2}{3}$, one finds that $\widehat{\beta} = \frac{2}{3}$ is the lowest belief about successors' patience that makes the all-wait path self-enforcing: when $\widehat{\beta} \geq \frac{2}{3}$ the agent expects successors will wait and the actual selves also prefer to wait until T. Thus $\widehat{\beta} = \frac{2}{3}$ is the threshold the hint suggests.

(2) Show that when $\widehat{\beta} < \frac{2}{3}$ the project is completed in period 0.

Sketch of argument. If $\widehat{\beta} < \frac{2}{3}$ the agent at $t = 0$ anticipates that successors are insufficiently patient and therefore does not expect them to postpone all the way to T. Consequently the planned continuation value is low enough that the present self prefers to do the project immediately rather than wait and risk lower outcomes. Since the present self actually acts with true $\beta = \frac{1}{2}$ but faces the immediate tradeoff $v - 1$ versus $\frac{1}{2}V_1$, and because $\widehat{\beta} < \frac{2}{3}$ implies V_1 is small, the inequality $v - 1 \geq \frac{1}{2}V_1$ holds, so the project is done at $t = 0$. Thus for $\widehat{\beta} < \frac{2}{3}$ immediate completion at period 0 is the unique equilibrium.

Problem 7 Inequality Aversion Model

(a) Standard assumptions on parameters

The usual parameter assumptions are:

$$\alpha_i \geq \beta_i \geq 0, \qquad \alpha_i < 1, \quad \beta_i < 1.$$

Intuition: people dislike being worse off more than they dislike being better off; both parameters are nonnegative and not so large as to make spite dominate own-payoff.

(b) Subgame perfect equilibrium

If both players are selfish, utility equals monetary payoff, the responder accepts whenever $s > 0$, since accepting gives $s > 0$ while rejecting yields 0, and is indifferent at $s = 0$. By backward induction, the proposer chooses the smallest s that will be accepted (a limit $\varepsilon \to 0$), so the SPNE outcome is: proposer offers $s \approx 0$ and keeps nearly the whole pie, or exactly $s = 0$ if accept-at-zero is allowed.

(c) Subgame perfect equilibrium with Fehr-Schmidt responder

We compute responder's acceptance condition using U_2. For an offered s the monetary payoff to player 2 is $x_2 = s$, to player 1 is $x_1 = 1 - s$. Two cases:

Case 1: $s \leq \frac{1}{2}$, responder does not get more than proposer. Then $x_1 \geq x_2$ and

$$U_2(s, 1-s) \;=\; s \;-\; \beta_2\,(x_1 - x_2) \;=\; s \;-\; \beta_2(1-2s).$$

The responder accepts iff $U_2(s, 1-s) \geq 0$, i.e.

$$s - \beta_2(1-2s) \geq 0 \quad\Longleftrightarrow\quad s(1+2\beta_2) \geq \beta_2 \quad\Longleftrightarrow\quad s \geq \frac{\beta_2}{1+2\beta_2}.$$

Case 2: $s \geq \frac{1}{2}$, responder gets at least as much as proposer. Then $x_2 \geq x_1$ and

$$U_2(s, 1-s) \;=\; s \;-\; \alpha_2\,(x_1 - x_2)_+ \;=\; s \;-\; \alpha_2 \cdot 0 \;=\; s.$$

So for $s \geq \frac{1}{2}$ the responder always has $U_2 \geq 0$ and accepts.

Minimal acceptable offer and proposer's best response. Combine the two cases. The minimal s that the responder will accept is

$$s^* \;=\; \min\Big\{\frac{\beta_2}{1+2\beta_2},\ \tfrac{1}{2}\Big\} = \frac{\beta_2}{1+2\beta_2},$$

because $\dfrac{\beta_2}{1+2\beta_2} < \frac{1}{2}$ for all $\beta_2 > 0$. If $\beta_2 = 0$ then $s^* = 0$.

Anticipating this, the proposer offers s^* and obtains payoff $1 - s^*$. Thus the SPNE is:

$$\text{Proposer offers } s^* = \frac{\beta_2}{1+2\beta_2}, \quad \text{Responder accepts.}$$

(d) Can Fehr-Schmidt explain dictator-game behavior?

In the dictator game the proposer (dictator) unilaterally chooses s and the responder has no veto. The dictator's choice maximizes her own Fehr–Schmidt utility $U_1(1-s,s)$. For the dictator (player 1) with parameters (α_1, β_1),

$$U_1(1-s,s) \;=\; \begin{cases} (1-s) - \beta_1(1-2s), & s \le \frac{1}{2}, \\ (1-s) - \alpha_1(2s-1), & s \ge \frac{1}{2}. \end{cases}$$

Maximizing this w.r.t. s yields:

$$\frac{dU_1}{ds} = \begin{cases} -1 + 2\beta_1, & s < \frac{1}{2}, \\ -1 - 2\alpha_1, & s > \frac{1}{2}. \end{cases}$$

Thus

- If $\beta_1 < \frac{1}{2}$, then $\frac{dU_1}{ds} < 0$ for $s < \frac{1}{2}$ so the dictator prefers s as small as possible (i.e. give 0).
- If $\beta_1 > \frac{1}{2}$, the dictator has an interior incentive to increase s up to $s = \frac{1}{2}$. For $s > \frac{1}{2}$ the derivative is negative (since $\alpha_1 \ge 0$), so the optimum is at $s = \frac{1}{2}$.

Conclusion: Fehr-Schmidt predicts positive giving by dictators only if their advantageous-inequality aversion β_1 is sufficiently large ($> 1/2$). With standard parameter calibrations ($\beta \approx 0.2$-0.4), the model predicts nearly zero offers, which contradicts experimental regularities where dictators often give a nontrivial positive fraction.

Therefore: Fehr–Schmidt can *partly* explain some pro-social transfers in dictators (if one allows sufficiently large β_1 or heterogeneous types), but with standard parameter ranges it underpredicts the observed generosity. Other motives (pure altruism, social norms, warm-glow, concerns for fairness beyond simple inequality aversion, or experiments inducing social preferences) may be needed to match empirical dictator behavior.

AI Report

In **Problem 2**, the paragraph **(Optional) Expected-utility calculus.** uses `ChatGPT-5`.

In **Problem 5**, the text is generated by `ChatGPT-5` and modified by the author.

In **Problem 6**, The discussion incorporates insights from `ChatGPT-5`, making the context very long.

In **Problem 7**, `ChatGPT-5` is used to explain **(d) Can Fehr-Schmidt explain dictator-game behavior?**.

English-Chinese translation is completed by `ChatGPT-5`.