Behavioral Economics HW2

Xi Xiang 231098288

March 17, 2026

1 Problem 1 Two-Person Beauty Contest Game

(a) Nash equilibrium and weakly dominant strategy

Let the two players choose numbers $x_1, x_2 \in [0, 100]$. The winning number is

$$T = \frac{2}{3} \cdot \frac{x_1 + x_2}{2} = \frac{x_1 + x_2}{3}.$$

A Nash equilibrium is a profile (x_1^*, x_2^*) such that each x_i^* is a best response to the other player's choice.

Suppose player 2 chooses x_2. Player 1 wants to choose x_1 as close as possible to

$$T = \frac{x_1 + x_2}{3}.$$

This implies that at equilibrium we must have

$$x_1 = \frac{x_1 + x_2}{3}.$$

By symmetry, also

$$x_2 = \frac{x_1 + x_2}{3}.$$

Solving these equations yields

$$x_1 = x_2 = 0.$$

Hence, the unique Nash equilibrium is

$$(x_1^*, x_2^*) = (0, 0).$$

Regarding dominance: choosing 0 weakly dominates any higher number. For any belief about the opponent's choice, choosing a smaller number moves the target downward and weakly improves the chance of being closer to the target. Therefore, 0 is a weakly dominant strategy for each player.

(b) L0 and L1 strategies

An $L0$ player chooses randomly or according to a naïve heuristic, without strategic reasoning. A common assumption is that the $L0$ player chooses uniformly on $[0, 100]$, so the expected choice is

$$\mathbb{E}[x^{L0}] = 50.$$

An $L1$ player best responds to the belief that the other player is $L0$. If the opponent's expected choice is 50, then the expected target is

$$T = \frac{2}{3} \cdot \frac{50 + x_1}{2}.$$

The $L1$ player chooses x_1 equal to the expected target, which leads to

$$x^{L1} = \frac{2}{3} \cdot 50 = \frac{100}{3} \approx 33.3.$$

Thus, the $L1$ optimal strategy is to choose approximately 33.

(c) Comparison with the N-person game ($N > 2$)

Yes, we would expect chosen numbers to be lower in the 2-person beauty contest game than in the N-person game with $N > 2$.

With fewer players, each individual has a larger influence on the mean. This strengthens the incentive to reason strategically and to anticipate the other player's reasoning steps. As a result, iterative elimination of dominated strategies is more transparent and more likely to push choices toward the Nash equilibrium at 0. In contrast, in larger groups each individual's impact on the mean is smaller, which tends to sustain higher choices.

Problem 2 Market Entry Game and Level-k Model

(a) Symmetric mixed-strategy equilibrium

There are three identical firms. Each firm chooses whether to *Enter* (E) or *Stay Out* (O). Payoffs are:

- If exactly 1 or 2 firms enter: entrants get 9, non-entrants get 8.
- If all 3 firms enter: all entrants get 0.

Let each firm enter with probability p in a symmetric mixed-strategy equilibrium.

Consider a representative firm. The probability that the other two firms generate:

- 0 entrants: $(1-p)^2$
- 1 entrant: $2p(1-p)$
- 2 entrants: p^2

Expected payoff from entering is

$$\pi_E = 9\big[(1-p)^2 + 2p(1-p)\big] + 0 \cdot p^2 = 9(1-p^2).$$

Expected payoff from staying out is always

$$\pi_O = 8.$$

In equilibrium, firms are indifferent:

$$9(1-p^2) = 8 \quad \Rightarrow \quad p^2 = \frac{1}{9} \quad \Rightarrow \quad p = \frac{1}{3}.$$

Thus, the unique symmetric mixed-strategy equilibrium has each firm entering with probability

$$p^* = \frac{1}{3}.$$

Distribution of total entrants. With $p = \frac{1}{3}$, the number of entrants N is binomial with parameters $(3, \frac{1}{3})$:

$$\Pr(N=0) = \left(\frac{2}{3}\right)^3 = \frac{8}{27},$$

$$\Pr(N=1) = 3 \cdot \frac{1}{3}\left(\frac{2}{3}\right)^2 = \frac{12}{27},$$

$$\Pr(N=2) = 3 \cdot \left(\frac{1}{3}\right)^2 \frac{2}{3} = \frac{6}{27},$$

$$\Pr(N=3) = \left(\frac{1}{3}\right)^3 = \frac{1}{27}.$$

The expected number of entrants is

$$\mathbb{E}[N] = 3 \cdot \frac{1}{3} = 1.$$

(b) Level-k behavior

Each firm's type is drawn independently:

$$\Pr(L1) = \Pr(L2) = \frac{1}{2}.$$

L1 behavior. An $L1$ firm believes each other firm enters with probability $\frac{1}{2}$, independently.

Expected payoff from entering:

$$\pi_E^{L1} = 9\left[1 - \left(\frac{1}{2}\right)^2\right] = 9 \cdot \frac{3}{4} = 6.75.$$

Since $6.75 < 8$, $L1$ firms choose *Stay Out*.

L2 behavior. An $L2$ firm best responds to $L1$. Since $L1$ firms stay out with probability 1, an $L2$ firm expects zero other entrants. Then

$$\pi_E^{L2} = 9 > 8 = \pi_O,$$

so $L2$ firms choose *Enter*.

Actual distribution of entrants. Each firm enters if and only if it is type $L2$. Hence each firm enters with probability $\frac{1}{2}$. The total number of entrants is binomial $(3, \frac{1}{2})$:

$$\Pr(N = 0) = \frac{1}{8},$$
$$\Pr(N = 1) = \frac{3}{8},$$
$$\Pr(N = 2) = \frac{3}{8},$$
$$\Pr(N = 3) = \frac{1}{8}.$$

The expected number of entrants is

$$\mathbb{E}[N] = 3 \cdot \frac{1}{2} = 1.5,$$

which is closer to the ex post optimal number 2 than the equilibrium value 1 from part (a).

Moreover,

$$\Pr(N=2) = \frac{3}{8} > \frac{6}{27},$$

so the probability of exactly two entrants is higher than in the mixed-strategy equilibrium.

(c) Sophisticated players

Now types are distributed as:

$$\Pr(L1) = \frac{1}{2}, \quad \Pr(L2) = \frac{1}{4}, \quad \Pr(S) = \frac{1}{4}.$$

$L1$ stays out and $L2$ enters, as before. A Sophisticated player best responds to this known type distribution.

Each other firm enters with probability $\frac{1}{4}$. The probability that both other firms enter is $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$. Thus,

$$\pi_E^S = 9\left(1 - \frac{1}{16}\right) = \frac{135}{16} > 8.$$

Hence Sophisticated players enter.

(d) Vanishing fraction of Sophisticated players

Let $\Pr(S) = \varepsilon \approx 0$ and $\Pr(L2) = \frac{1}{2} - \varepsilon$, with $\Pr(L1) = \frac{1}{2}$ fixed.

Each other firm enters with probability $\frac{1}{2} - \varepsilon$. The probability both others enter is $(\frac{1}{2} - \varepsilon)^2 \approx \frac{1}{4}$. Then

$$\pi_E^S \approx 9\left(1 - \frac{1}{4}\right) = 6.75 < 8,$$

so Sophisticated players optimally stay out. Therefore, when Sophisticated players are sufficiently rare, they do not enter.

AI Report

English-Chinese translation is completed by `ChatGPT-5`.