Problem Set 1

Please submit the answers to luyunfeng@nju.edu.cn in pdf version before ***Dec 18***. The Answers could be written by either English or Chinese.

1. *Heuristics and biases*

Suppose that, in the course of a regular check-up, a doctor discovers that the patient has a potentially cancerous lesion. Most lesions are benign (non-cancerous), say 99%. The doctor orders an x-ray just in case. In laboratory tests on malignant (cancerous) lesions, the x-ray returns positive (cancer-affirming) results 79.2% of the time and negative results 20.8% of the time. In laboratory tests on benign lesions, the x-ray returns positive results only 9.6% of the time and negative results 90.4% of the time.

(a) The patient's x-ray comes back positive. What is the probability that the patient has cancer?

(b) Suppose that the doctor calculates the probability that the patient has cancer without regard to the base rate of cancer in the population – that is, the doctor uses Bayes' Rule but assumes that cancerous and non-cancerous lesions are equally likely. What mistaken conclusion will the doctor draw from the test? How is this mistake an example of representativeness?

(c) Explain why it is important in these situations to have hospital procedures that require additional tests to be performed before a patient undergoes treatment.

2. *BDM mechanism*

   The following describes the Becker-DeGroot-Marschak procedure for eliciting the valuation of an object (let's say a teapot) in an experiment. Kahneman, Knetsch, and Thaler (1990) used this mechanism to elicit the valuations for mugs. For convenience, let's assume that no-one should value a teapot over $1000.

1) Ask the subject to name their valuation of the teapot
2) Draw a random number x between 0 and 1000
3) If the x is above the subject's valuation then nothing happens
4) If the x is below their valuation, then they get the teapot and pay x (NOT their valuation)

   Show that this procedure is incentive compatible: i.e. that the best thing that a subject can do is announce their true valuation. Notice that the BDM mechanism is equivalent to a Vickrey auction against an unknown bidder.

3. *The Allais Paradox*

   In 1953, Maurice Allais proposed the following thought-experiment. You must make a choice between Gamble A and Gamble B (you can interpret these dollar amounts as final wealth levels):

   Gamble A: $1 million for sure

   Gamble B: $1 million with probability 0.89, $5 million with probability 0.10, and $0 with probability 0.01.

   Which would you choose?

Next, you must make a choice between Gamble C and Gamble D:
Gamble C: \$1 million with probability 0.11, and \$0 with probability 0.89
Gamble D: \$5 million with probability 0.10, and \$0 with probability 0.90
Which would you choose?

Most people choose Gambles A and D. Explain why (no matter what utility function a person has) this pattern of choices violates expected utility theory.

4. ***Prospect theory***
Let $U(x) = \begin{cases} x^{\theta}, & x \geq 0 \\ -\lambda(-x)^{\theta}, & x < 0 \end{cases}$ with $\theta = 0.5$ and $\lambda = 2$. $w^{+}(p) = p^2$, and $w^{-}(p) = p$.
Determine the decision weights of each outcome, and the PT value of the prospect L = (10%, 36; 20%, 25, 30%, -9; 40%, -16).

5. *Four-fold risk pattern*
Please briefly state that why the prospect theory can explain the four-fold risk pattern.

6. ***Doing it now or later***

(a) Consider a quasi-hyperbolic Naif with δ = 1 and β = ½ but his own estimate of β is $\hat{\beta}$ = 1. Time is indexed $t \in \{0,1,2,\ldots\}$. The Naif must finish a project by a deadline T < ∞ at the latest. In period t the project costs $(\frac{3}{2})^t$ to finish, and the value is v if the project is finished. (There is no time discounting; just the increasing cost of doing it later.) Commitment is impossible. When will the Naif do the project?

(b) Now consider a quasi-hyperbolic sophisticated agent with δ = 1 and $\beta = \hat{\beta} = \frac{1}{2}$, with everything else as in part (a). Recall that in this dynamic setting, an agent's behavior must be characterized by a complete contingent plan (or strategy). Prove the following two claims:
  (1) If T is an even number, then a Sophisticate will do the project in any even period (that is, if he has not already done it) but not in any odd period.
  (2) If T is an odd number, then a Sophisticate will do the project in any odd period (if he has not already done it) but not in any even period.

(c) Now consider a quasi-hyperbolic partially-naive agent with δ = 1, β = ½, but his own estimate of β is $\hat{\beta} \in (\beta, 1)$. That is, the agent incorrectly believes that her/his future selves will have beta parameter $\hat{\beta} > \beta$, when in fact it will be β. Assume T is even, with everything else as before.
  (1) Solve for the lowest value of $\hat{\beta}$ for which there is an equilibrium in which the agent does not do the project until period T. (Hint: Consider the case $\hat{\beta} = \frac{2}{3}$ and show that this is the key threshold value.)
  (2) Show that when $\hat{\beta} < \frac{2}{3}$, the project is completed in period 0.

7. *Inequality aversion model*

Consider the standard ultimatum game. Player 1 is the proposer and Player 2 is the responder, whose Fehr–Schmidt preference parameters are commonly given by (α, β), such as

$$U_1(x_1, x_2) = \begin{cases} x_1 - \alpha_1(x_2 - x_1)\ if\, x_2 \geq x_1 \\ x_1 - \beta_1(x_1 - x_2)\ if\, x_1 \geq x_2 \end{cases}$$

The proposer offers a share s ∈ [0, 1] that is observed by the responder who chooses to accept or reject. If the offer is accepted, the responder gets s, the proposer gets 1 – s; if the offer is rejected, both players get nothing.

(a) Please state the assumptions of the Fehr-Schmidt utility function parameters.
(b) Please show the subgame perfect equilibrium of this game with the standard utility.
(c) Please show the subgame perfect equilibrium with the Fehr-Schmidt utility function (for simplicity, assume the proposer knows the preference of the responder).
(d) Can this inequality aversion model explain the experimental results in the dictator game? Why?