

Lecture 2

Time Value of Money

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2 Sep 2025

Discussion

Discussion: if your friends or relatives borrows ¥1,000 from you, how much should you ask him to repay in one year?

传统思维：大家都是亲戚朋友，碍于面子，不好意思收取利息。
这节课，我们会从财务的角度看待这个问题。

Future Value

Future Value (*FV* 终值): The total amount due at the end of the investment.

- If you put \$10,000 in the bank at 5-percent interest for one year, your wealth would grow to:

$$10,000 \times (1+0.05) = \$ 10,500$$

Present Value



Present Value (*PV* 现值): The amount need to set aside today to meet the promised payment in the future.

- If you were to be promised \$10,000 due in one year when interest rates are 5-percent, your investment would be worth in today's dollars :

$$\$10,000/(1+0.05)= \$9,523.81$$

Simple vs Compound interest



- **Simple interest (单利)**: do not re-invest the interest into next period
 - Invest \$1 now and get $1 + 2 \times r$ in two years
- **Compound interest (复利)**: re-invest the interest into next period
 - Invest \$1 now and get $1 + r$ in one year, and reinvest $1 + r$ in the second year and finally get $(1 + r)^2$

Time Value of Money (货币的时间价值)



- The PV and FV can be generalized as

$$FV = PV \times (1 + r)^T \quad \text{or} \quad PV = FV / (1 + r)^T$$

- T is the number of periods over which the cash is invested
- r is the appropriate interest rate or discount rate
- **Compound interest** is assumed
- **Present value (or discount) factor:** $1/(1+r)^T$ always < 1
- If we know any three of the four factors, we can compute the last one

Time Value of Money

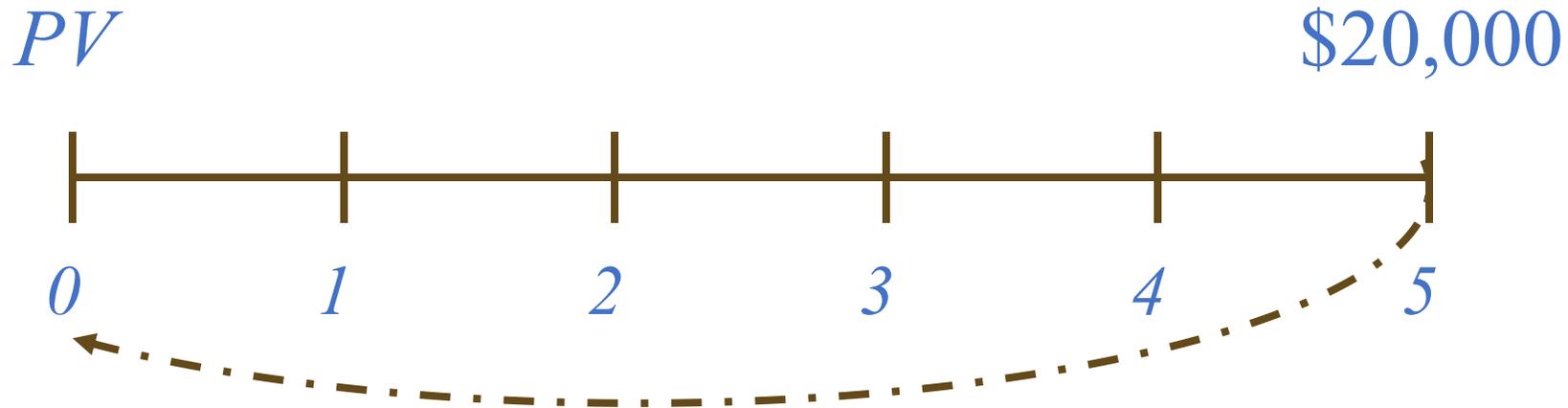
How to understand this concept?

- One dollar today is worth more than one dollar tomorrow.
- If your friend borrows ¥1,000 from you, how much should you ask him to repay in one year?
 - After today's class, you should ask more than ¥1,000

Draw a Time Line



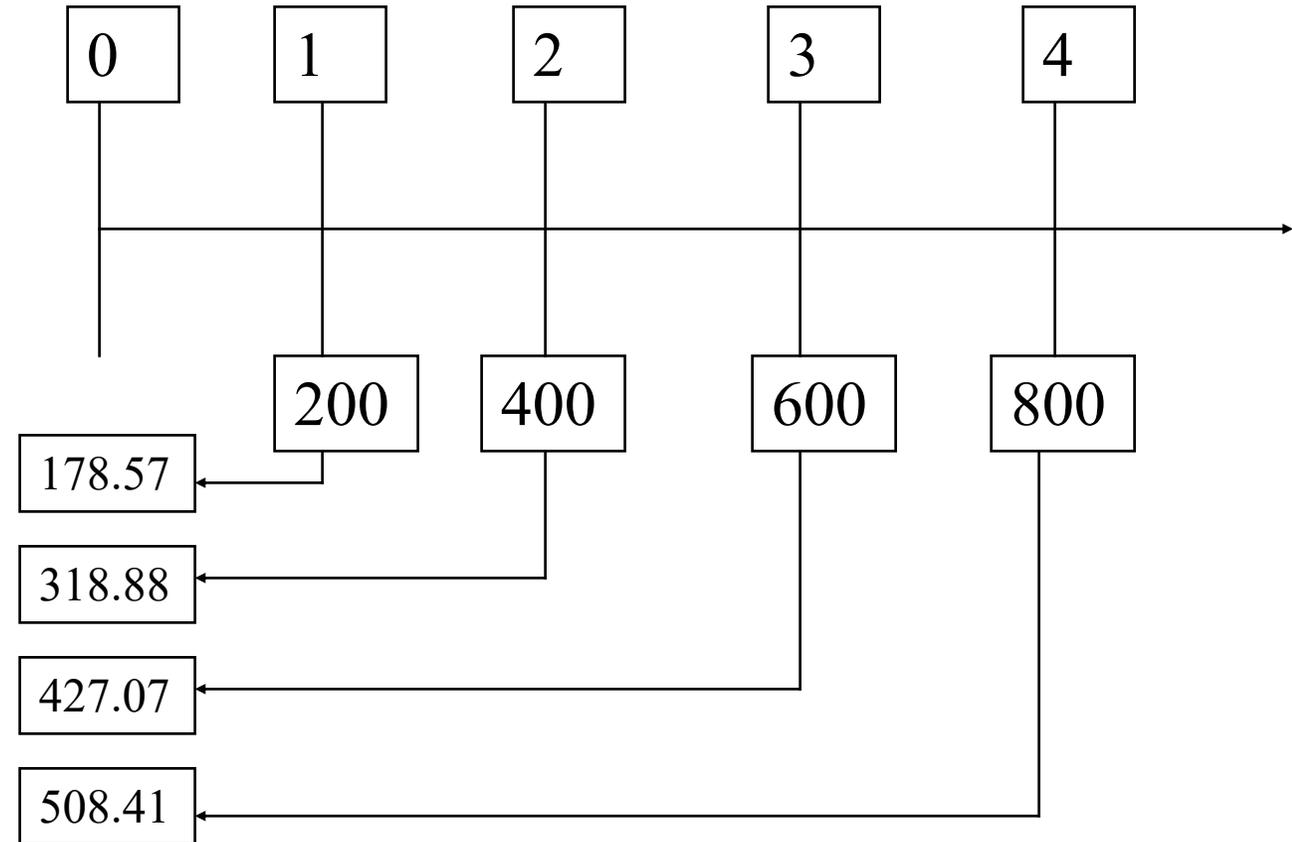
How much would an investor have to set aside today in order to have \$20,000 five years from now if the current rate is 15%?



$$\$9,943.53 = \frac{\$20,000}{(1.15)^5}$$

Multiple Periods

Consider an investment that pays \$200 one year from now, with cash flows increasing by \$200 per year through year 4. If the interest rate is 12%, what is the present value of this stream of cash flows?



Spreadsheet Calculation

Spreadsheet calculation is widely used in financial industry. Be familiar with it if you want to find a job or intern in an investment bank or as a financial analyst!

To Find

Enter This Formula

Future value

= FV (rate,nper,pmt,pv)

Present value

= PV (rate,nper,pmt,fv)

Discount rate

= RATE (nper,pmt,pv,fv)

Number of periods

= NPER (rate,pmt,pv,fv)

- The interest rate should be entered as a decimal (0.05 instead of 5)
- Put a negative sign on the present value as if it is a cash outflow.

Spreadsheet Calculation: Example

- Practice: Calculate the number of periods

	A	B	C	D	E	F	G	H
1								
2	Using a spreadsheet for time value of money calculations							
3								
4	If we invest \$25,000 at 12 percent, how long until we have \$50,000? We need to solve							
5	for the unknown number of periods, so we use the formula NPER(rate, pmt, pv, fv).							
6								
7	Present value (pv):	\$25,000						
8	Future value (fv):	\$50,000						
9	Rate (rate):	.12						
10								
11	Periods:	6.1162554						
12								
13	The formula entered in cell B11 is =NPER(B9,0,-B7,B8); notice that pmt is zero and that pv							
14	has a negative sign on it. Also notice that rate is entered as a decimal, not a percentage.							

Web Financial Calculator



金融计算器



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[金融计算器](#) 凯利公式计算器 凯利公式(Kelly Formula)计算器 内部收益率(IRR)计算器 能使投资项目净现值等于零时的折现率 现金流折现(DCF)计算器 采用多阶段增长模型 简单...

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金融计算器

凯利公式计算器

凯利公式(Kelly Formula)计算器

内部收益率(IRR)计算器

能使投资项目净现值等于零时的折现率

现金流折现(DCF)计算器

采用多阶段增长模型

简单利息计算器

把本金, 利率和时间段相乘来计算

复利计算器

可按照日, 星期, 月或者年来计算复利

零存整取存款计算器

周期性存入固定数量金额

信用卡还款计算器

开发中...

股票计算器

在线计算股票盈亏利率

股票买卖计算器

股票期望盈利利率卖出价格计算器

净现值计算器

简化净现值的计算

期值 (未来价值) 计算器

计算未来价值

修正内部收益率计算器

和内部收益率(IRR)接近的投资分析方...

折旧计算器

直线折旧和年数总和折旧

投资计算器

计算初始投资, 投资回报率, 周期投...

加法机计算器

开发中...

外汇汇率换算

各种汇率换算

股票大全

沪深A股、港股、美股名称代码大全

投资计算器

计算以下任一种情况 - 初始投资, 投资回报率, 周期投入金额, 和投资总收益。

我想计算:

投资目标
 投资回报率
 定期投入金额
 投资期长度
 初始投资数量

初始投资 (¥):

25,000

定期投入 (¥):

0

定期频率:

每年

投资回报率 (%):

12

投资期长度 (年):

7

复利计算:

每年

投资目标 (¥):

50,000

提交

重置

Compounding periods



- So far, we have assumed compounding and discounting occurs yearly. Sometimes, it may occur more frequently
- Compounding an investment m times a year for T years provides for future value of wealth:

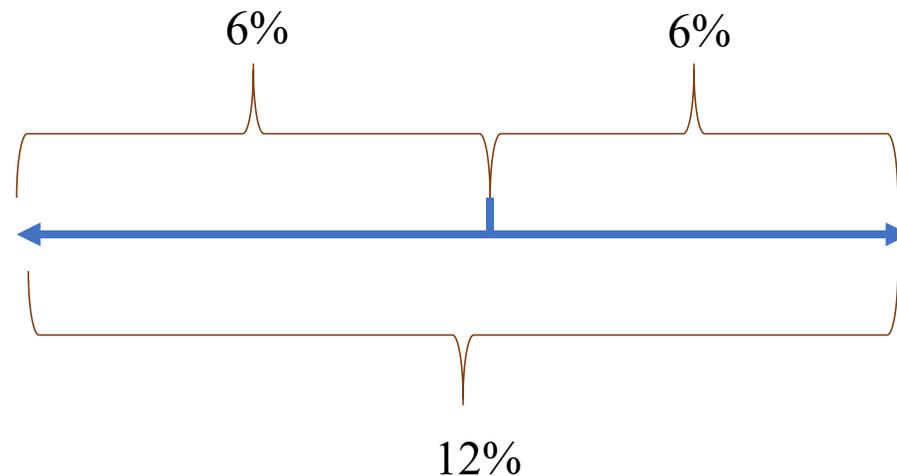
$$FV = C_0 \times \left(1 + \frac{r}{m}\right)^{m \times T}$$

- Always remember, r is a annual interest rate!

Compounding periods

- If you invest \$50 for 3 years at 12% compounded semi-annually, your investment will grow to:

$$FV = \$50 \times \left(1 + \frac{0.12}{2}\right)^{2 \times 3} = \$50 \times (1.06)^6 = \$70.93$$



Always remember: 12% is an annual rate.

Effective Annual Rate



- Effective Annual Rate (EAR) = $(1 + \frac{r}{m})^m - 1$
- The Effective Annual Rate (EAR) of interest is the annual rate that would give us the same end-of-investment wealth after 3 years:

$$\$50 \times (1 + \mathbf{EAR})^3 = \$70.93 \quad \mathbf{EAR} = \left(\frac{\$70.93}{\$50} \right)^{1/3} - 1 = .1236$$

12.36% Compounded annually is the same as investing at 12% compounded semi-annually.

Effective Annual Rate

$$\text{EAR} = \left(1 + \frac{r}{m}\right)^m - 1$$

Key Takeaway:

- If $m > 1$, then $\text{EAR} > r$
- Remember: r is the quoted interest rate, which is always an annualized rate.
- Which is better?
 - Semiannual compound at 1.9%
 - 余额宝：7日年化利率2%

Applications

- 余额宝/零钱通利息计算
- Perpetuity: A constant stream of cash flows that lasts forever
- Annuity: A stream of constant cash flows that lasts for a fixed number of periods
- Amortized loan: repayment of both principal and interest over time.

余额宝

余额宝：存款1万元，7日年化利率2%

- 1年后我能获得多少利息？
- 7日后我能获得多少利息？

一年后， $FV = 10000 \times (1 + 2\%) = 10200$

七日后， $FV = 10000 \times \left(1 + \frac{2\%}{365}\right)^7 = 10003.84$

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复利计算器

存款 (¥):
10000

年利率 (%):
2

年限:
1

复利计算: 每天

提交 重置

报告

Day	初始存款(¥)	利息(¥)	总计利息(¥)	本金+利息(¥)
1	\$10,000.00	\$0.55	\$0.55	\$10,000.55
2	\$10,000.55	\$0.55	\$1.10	\$10,001.10
3	\$10,001.10	\$0.55	\$1.64	\$10,001.64
4	\$10,001.64	\$0.55	\$2.19	\$10,002.19
5	\$10,002.19	\$0.55	\$2.74	\$10,002.74
6	\$10,002.74	\$0.55	\$3.29	\$10,003.29
7	\$10,003.29	\$0.55	\$3.84	\$10,003.84

Discussion

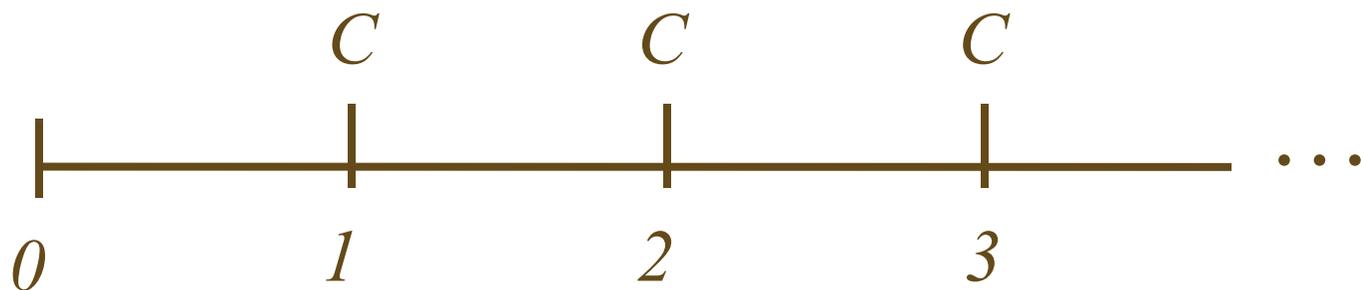
Back to the beginning question: if your friends or relatives borrows ¥1,000 from you, how much should you ask him to repay in one year?

你应该对他/她说：我本可以将这笔钱放到余额宝，赚取一定利息。借钱给你是帮你应对临时的困难，但不能让我自身利益受损。

Perpetuity (永续年金)

名师讲解

A constant stream of cash flows that lasts forever

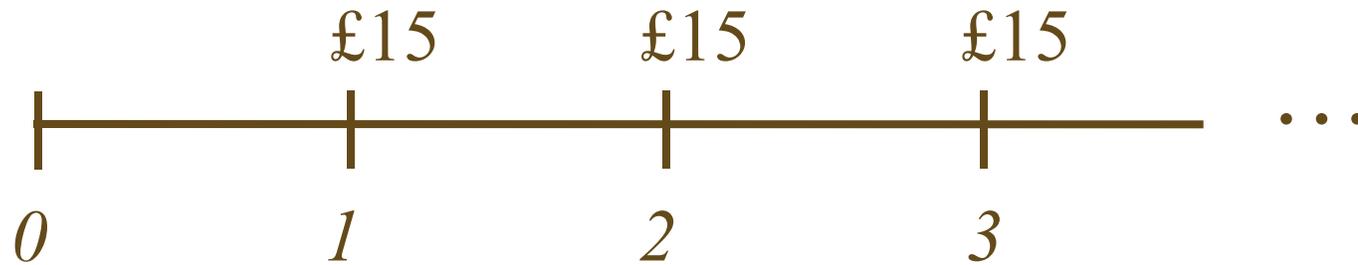


$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \dots$$

$$PV = \frac{C}{r}$$

Perpetuity: Example

What is the value of a Treasury bond that promises to pay \$15 every year for ever? The interest rate is 10-percent.



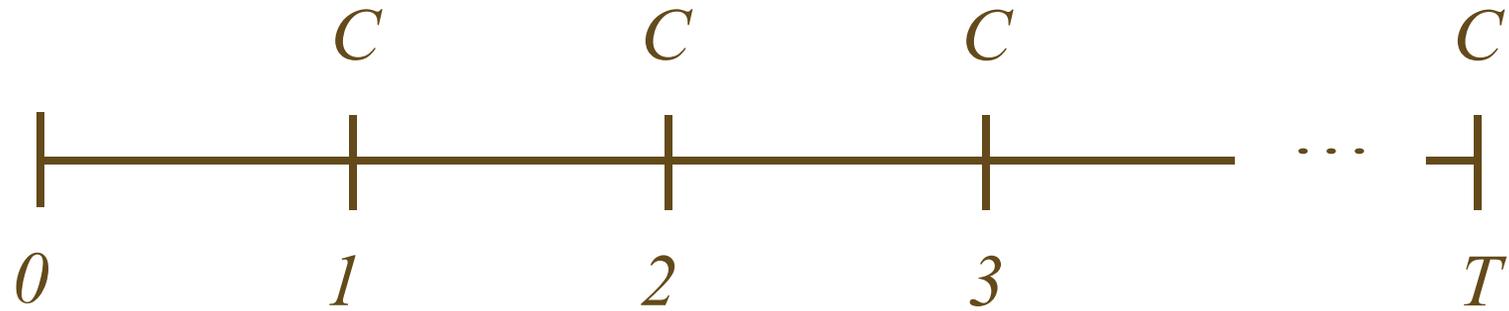
$$PV = \frac{\pounds 15}{.10} = \pounds 150$$

Annuity (年金)



名词解释

A constant stream of cash flows with a fixed maturity

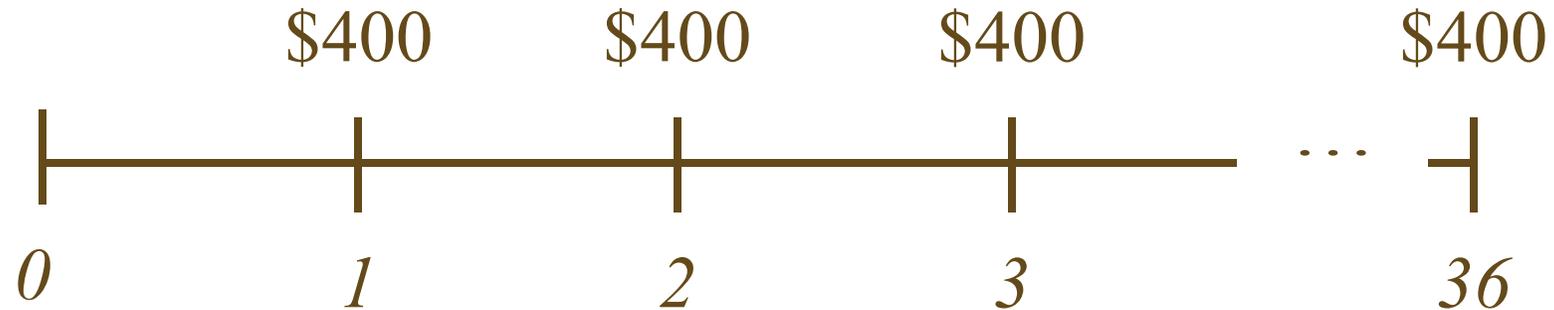


$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \dots + \frac{C}{(1+r)^T}$$

$$PV = \frac{C}{r} \left[1 - \frac{1}{(1+r)^T} \right]$$

Annuity

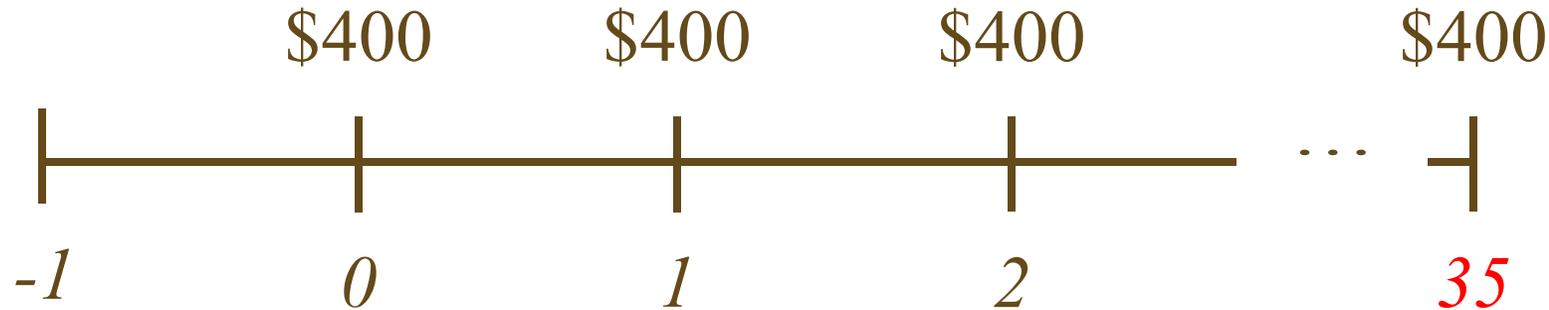
If you can afford a \$400 monthly car payment, how much car can you afford if interest rates are 7% on 36-month loans?



$$PV = \frac{\$400}{.07/12} \left[1 - \frac{1}{(1 + .07/12)^{36}} \right] = \$12,954.59$$

Annuity Due (即付年金)

The stream of cash flows happen at the beginning of each period.



First, calculate the PV at date -1 , then compound $1+r$ to date 0 .

$$12954.59 \times (1 + 0.07/12) = 13029.56$$

Installment Payment (分期付款)



RMB 333/月 (24 期) 或 RMB 7,999

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333*24月 or 7999?

余额宝七日年化利率：1.38%

$$PV = \frac{\$333}{.0138/12} \left[1 - \frac{1}{(1 + .0138/12)^{24}} \right]$$

$$= \$7878$$

Choose installment, save \$121

Installment Payment (分期付款)

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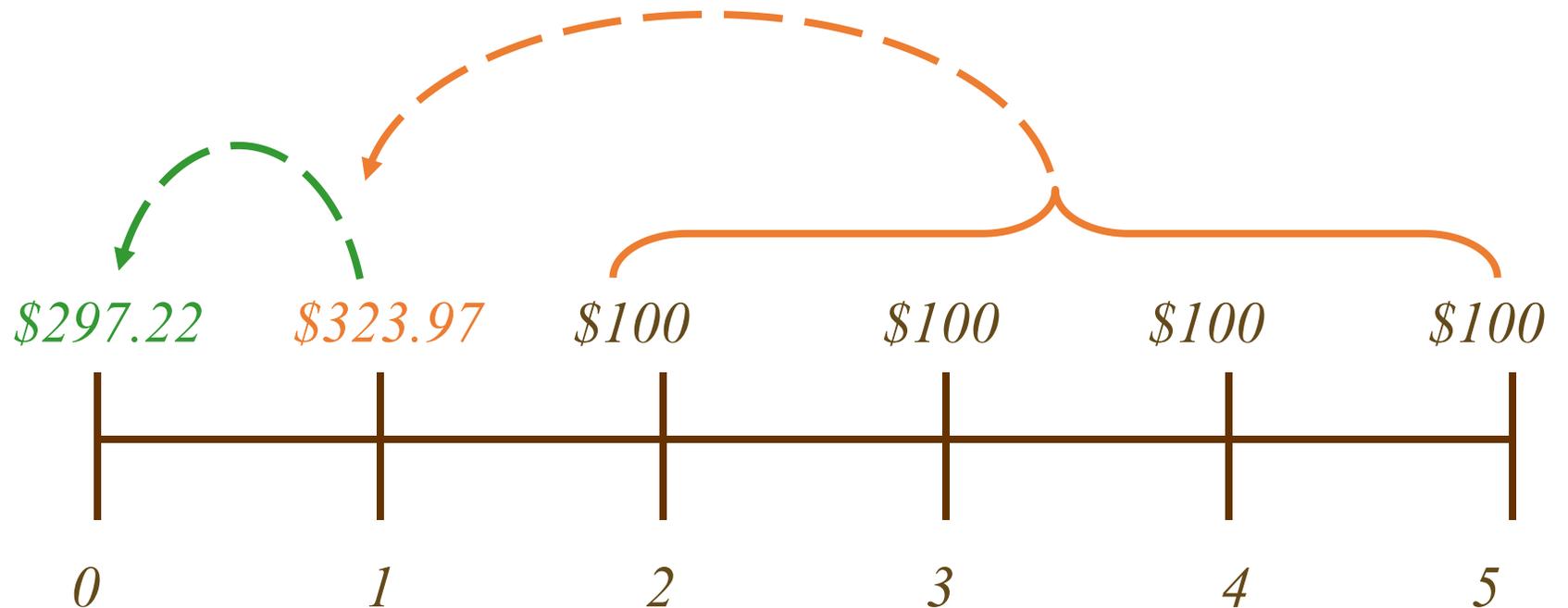
Delayed Annuity (递延年金)



What is the present value of a four-year annuity of \$100 per year that makes its first payment two years from today if the discount rate is 9%?

$$PV_1 = \sum_{t=1}^4 \frac{\$100}{(1.09)^t} = \frac{\$100}{(1.09)^1} + \frac{\$100}{(1.09)^2} + \frac{\$100}{(1.09)^3} + \frac{\$100}{(1.09)^4} = \$323.97$$

$$PV_0 = \frac{\$323.97}{1.09} = \$297.22$$



Loan Amortization (贷款摊销)



- **Pure Discount Loans** are the simplest form of loan. The borrower receives money today and repays a single lump sum (principal and interest) at a future time. 一次性还本付息)
- **Interest-Only Loans** require an interest payment each period, with full principal due at maturity. (月付利息到期还本)
- **Amortized Loans** require repayment of principal (本金) over time, in addition to required interest. (等额本息; 等额本金)

Equal principal payment (等额本金) ✗

Suppose a business takes out a \$5,000, five-year loan at 9%. The borrower pays the interest and a fixed amount of the principal.

First, calculate the fixed principal payment: $5,000/5=1000$

Amortization schedule:

Year	Beginning Balance	Total Payment	Interest Paid	Principal Paid	Ending Balance
1	\$5,000	\$1,450	\$ 450	\$1,000	\$4,000
2	4,000	1,360	360	1,000	3,000
3	3,000	1,270	270	1,000	2,000
4	2,000	1,180	180	1,000	1,000
5	1,000	1,090	90	1,000	0
Totals		\$6,350	\$1,350	\$5,000	

Equal Payment (等额本息) ✗

Suppose a business takes out a \$5,000, five-year loan at 9%. The borrower pays a fixed amount each year.

First, calculate the fixed payment as an annuity:

$$\begin{aligned} \$5,000 &= C \times \{ [1 - (1/1.09^5)] / .09 \} \\ &= C \times [(1 - .6499) / .09] \end{aligned}$$

This gives us:

$$\begin{aligned} C &= \$5,000 / 3.8897 \\ &= \$1,285.46 \end{aligned}$$

Equal Payment

Amortization schedule:

In the first year, the interest is $5,000 \times 0.09 = 450$

Thus, the principal paid = $1,285.46 - 450 = 835.46$

Ending balance = Beginning balance – principal paid = 4164.54

Year	Beginning Balance	Total Payment	Interest Paid	Principal Paid	Ending Balance
1	\$5,000.00	\$1,285.46	\$ 450.00	\$ 835.46	\$4,164.54
2	4,164.54	1,285.46	374.81	910.65	3,253.88
3	3,253.88	1,285.46	292.85	992.61	2,261.27
4	2,261.27	1,285.46	203.51	1,081.95	1,179.32
5	1,179.32	1,285.46	106.14	1,179.32	0.00
Totals		\$6,427.30	\$1,427.31	\$5,000.00	

Equal Principle payment

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- 货币时间价值
- 利率换算
- 房贷摊销
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- 债券计算器
- 统计计算器
- 普通计算器
- 关于秉文

房贷摊销

n

I %

PV

FV

还款方式 等额本息 等额本金

开始期数 结束期数

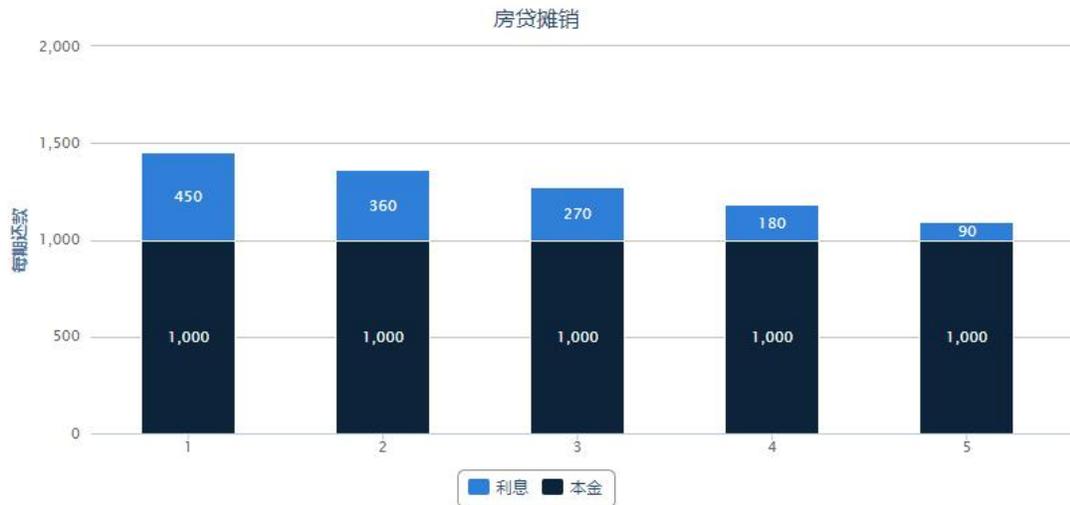
每期摊还额

累计偿还本金

累计偿还利息

剩余贷款本金

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Equal Payment

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货币时间价值

利率换算

房贷摊销

现金流

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统计计算器

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n

i %

PV

FV

还款方式 等额本息 等额本金

开始期数

结束期数

每期摊还额

累计偿还本金

累计偿还利息

剩余贷款本金

房贷摊销



Equal principal vs Fixed Payment

- 等额本金 vs 等额本息
- The total interest paid is higher under the fixed payment method. Therefore, the loan officer would typically recommend you the fixed payment method (等额本息)!
- Under the fixed principal-paid method, the total pay is higher in initial years and keeps declining as time passing by.

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