Lecture 7

Risk and Return

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Stock Return（股票回报率）

- The return of a stock can include
  - dividend
  - capital gain (change in market value)

- Percentage return=Dollar return/beginning market value

  =(Dividend + capital gain) / Beginning market value

  = **Dividend yield** + Capital gains yield

- **Holding Period Return（持有期回报率）**： the return that an investor would get when holding an investment over a period of T years:

$$HPR = (1 + R_1) \times (1 + R_2) \times \cdots \times (1 + R_T) - 1$$

Holding period return

Suppose your investment provides the following returns over a four-year period:

Year	*Return*
1	10%
2	-5%
3	20%
4	15%

Your holding period return =

$$= (1+R_1)\times(1+R_2)\times(1+R_3)\times(1+R_4)-1$$

$$= (1.10)\times(.95)\times(1.20)\times(1.15)-1$$

$$= .4421 = 44.21\%$$

Average returns

If we want to evaluate the performance of a fund manager over 20 years, we need to use average returns.

- **Arithmetic average（算术平均）**: return earned in an average period over multiple periods
- **Geometric average（几何平均）**： average compound return per period over multiple periods
- The geometric average will be less than the arithmetic average unless all the returns are equal.

Average returns: Example

Recall our earlier example:

Year	*Return*
1	10%
2	-5%
3	20%
4	15%

- Arithmetic average return
  =(10%-5%+20%+15%)/4
  =10%
- Geometric average return
  $=(1+10\%)(1-5\%)(1+20\%)(1+15\%)^{1/4}-1$
  $=9.58\% < 10\%$
- The fund manager made an average of 9.58% per year, realizing a holding period return of 44.21%.

Risk(风险)

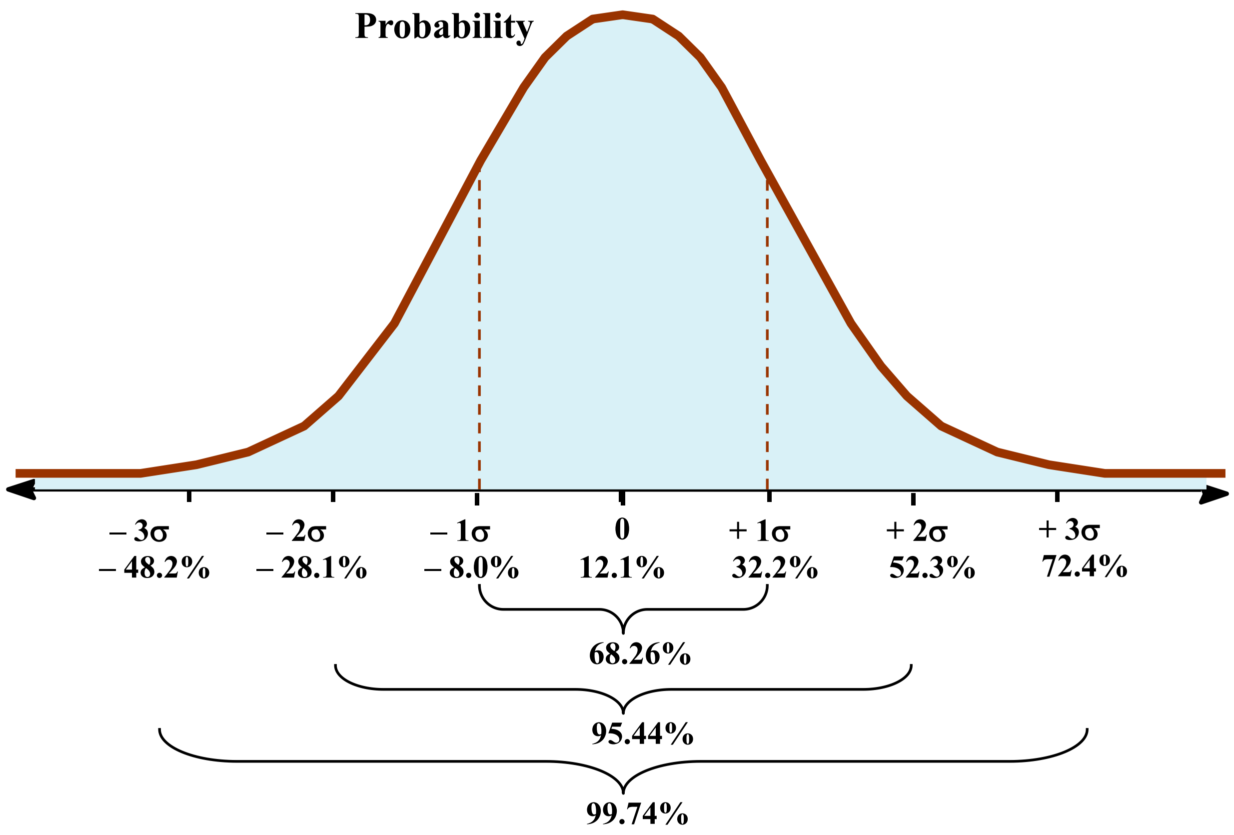
- There is no universally agreed-upon definition of risk. One way to understand it is how spread out the frequency distribution of return is.

- The measures of risk that we discuss are **variance** and **standard deviation**.

$$SD = \sqrt{VAR} = \sqrt{\frac{(R_1 - \overline{R})^2 + (R_2 - \overline{R})^2 + \cdots (R_T - \overline{R})^2}{T}}$$

Why variance measures risk?

We can assume the expected stock return follows a normal distribution. A larger standard deviation means a higher probability the return will deviate from the mean values.



The probability that a yearly return will fall within one standard deviation of the mean will be approximately 2/3.

Standard deviation: Example

Year	Actual Return	Average Return	Deviation from the Mean	Squared Deviation
1	.15	.105	.045	.002025
2	.09	.105	-.015	.000225
3	.06	.105	-.045	.002025
4	.12	.105	.015	.000225
Totals			.00	.0045

Variance = .0045 / 4= .0011 Standard Deviation = .0336

Risk Premium(风险溢价)

名词解释

- **Risk free rate**: the interest rate on risk-free assets, such as T-bill, money market fund, bank deposit.

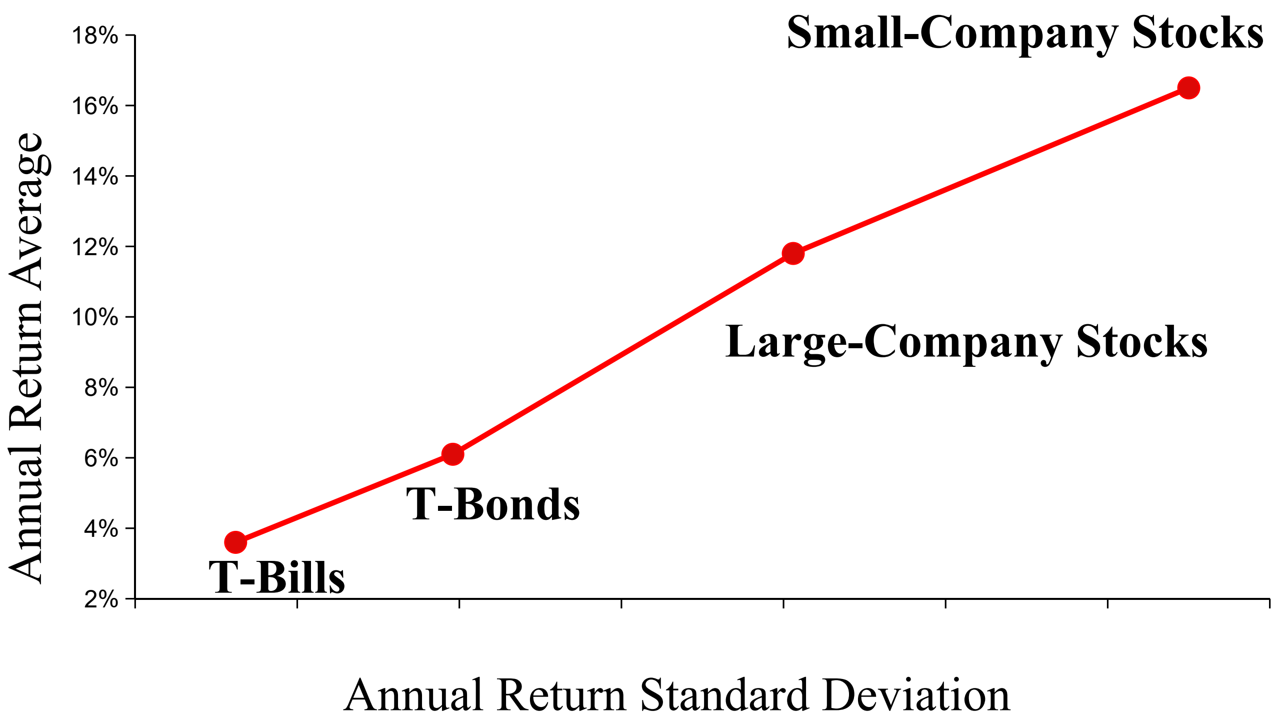
- **Risk premium** (excess return)= return – risk-free rate. It is the added return (over and above the risk-free rate) resulting from bearing risk. (Recall credit spread)

- **Equity risk premium**: average excess return on common stocks

  股票风险溢价

Risk-return tradeoff

Higher risk, higher return



中国银行保险监督管理委员会主席郭树清在第十届陆家嘴论坛上表示，在打击非法集资过程中，努力通过多种方式让人民群众认识到，高收益意味着高风险，收益率超过6%的就要打问号，超过8%的就很危险，10%以上就要准备损失全部本金。

Sharpe ratio

夏普比率 计算

- Sharpe ratio= Risk Premium/Standard deviation of returns
  = (Expect return – Risk free rate)/ S.D (Expected return)

- It measures the return generated when one undertakes one unit of risk. It reflects the tradeoff between risk and return.

- It can be used to evaluate the performance of financial assets, such as stock and bond. It can also be used to compare performance for fund managers.

- A higher Sharpe ratio indicates a superior performance

Risk and return: Two securities

Which stock should you buy?

Case 1:

Stock A: $E(R)=10\%$ $\sigma(R)=5\%$

Stock B: $E(R)=15\%$ $\sigma(R)=10\%$

Case 2:

Stock A: $E(R)=10\%$ $\sigma(R)=5\%$

Stock B: $E(R)=15\%$ $\sigma(R)=3\%$

Risk and return: Two securities

Let E(R) and σ *(R)* to be the expected return and its standard deviation on one single stock.

- **Covariance**: how returns on one security are related to another security.

$$Cov\,(R_1,\ R_2) = E\{[R_1\text{-E}(R_1)][R_2\text{-E}(R_2)]\}$$

- **Correlation coefficient**: $\rho = \frac{Cov\,(R_1, R_2)}{\sigma_1 \sigma_2}$

Correlation

- Unless ρ =0, the return of two stocks can have some co-movement. ρ varies from -1 to 1.
- ρ >0 positive correlation: one stock price increases, another tend to increase too. 美的 vs. 格力
- ρ <0 negative correlation: one stock price increases, another tend to decrease. For example, 新冠疫苗股 vs. 新冠药物股
- Perfect positive correlation: ρ=1; Zero correlation: ρ=0; Perfect negative correlation: ρ=-1

Security analysis: Example

Consider the following two risky asset world. There is a 1/3 chance of each state of the economy, and the only assets are a stock fund and a bond fund.

		Rate of Return	
Scenario	*Probability*	*Stock Fund*	*Bond Fund*
Recession	33.3%	-7%	17%
Normal	33.3%	12%	7%
Boom	33.3%	28%	-3%

Security analysis: Expected Return

Scenario	Stock Fund Rate of Return	Stock Fund Squared Deviation	Bond Fund Rate of Return	Bond Fund Squared Deviation
Recession	-7%	0.0324	17%	0.0100
Normal	12%	0.0001	7%	0.0000
Boom	28%	0.0289	-3%	0.0100
Expected return	11.00%		7.00%	
Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

$$E(r_S) = \tfrac{1}{3} \times (-7\%) + \tfrac{1}{3} \times (12\%) + \tfrac{1}{3} \times (28\%)$$

$$E(r_S) = 11\%$$

Security analysis: Variance

	Stock Fund		Bond Fund	
Scenario	**Rate of Return**	**Squared Deviation**	**Rate of Return**	**Squared Deviation**
Recession	-7%	0.0324	17%	0.0100
Normal	12%	0.0001	7%	0.0000
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Standard Deviation	14.3%		8.2%	

$$(-7\% - 11\%)^2 = .0324$$

Security analysis: Variance

	Stock Fund		Bond Fund	
Scenario	***Rate of Return***	***Squared Deviation***	***Rate of Return***	***Squared Deviation***
Recession	-7%	0.0324	17%	0.0100
Normal	12%	0.0001	7%	0.0000
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$$.0205 = \frac{1}{3}(.0324 + .0001 + .0289)$$

Security analysis: Covariance and Correlation

Scenario	Stock Deviation	Bond Deviation	Product	Weighted
Recession	-18%	10%	-0.0180	-0.0060
Normal	1%	0%	0.0000	0.0000
Boom	17%	-10%	-0.0170	-0.0057
Sum				-0.0117
Covariance				-0.0117

- Recall $Cov\,(R_1,\ R_2) = E\{[R_1\text{-E}(R_1)][R_2\text{-E}(R_2)]\}$
- $\rho = \frac{Cov\,(R_1, R_2)}{\sigma_1\sigma_2} = \frac{-0.0117}{(0.143)(0.082)} = -0.998$
- "Deviation" compares return in each state to the expected return. "Weighted" takes the product of the deviations multiplied by the probability of that state.

Portfolio analysis

- In the real world, a fund invest in many asset classes at the same time and form a **portfolio（投资组合）**. A simple case is: invest w_b in Bond and w_s in Stock (w_b+ w_s =1).

- **Portfolio expected return**: $E(r_P) = w_B E(r_B) + w_S E(r_S)$

- **Portfolio return variance**:

$$\sigma_P^2 = (w_B\sigma_B)^2 + (w_S\sigma_S)^2 + 2w_B w_S \boldsymbol{\rho_{BS}} \sigma_B \sigma_S$$

- ρ_{BS} is the correlation coefficient between the returns on stock and bond

- Diversification effect: 分散效应 Portfolio SD is less than the weighted average of the SD of the individual securities as long as ρ_{BS}<1.

Portfolio SD $= \sigma_P^2 = (w_B\sigma_B + w_S\sigma_S)^2 < SD^2(B+S)$

Portfolio analysis: Example

	Stock Fund		Bond Fund	
Scenario	***Rate of Return***	***Squared Deviation***	***Rate of Return***	***Squared Deviation***
Recession	-7%	0.0324	17%	0.0100
Normal	12%	0.0001	7%	0.0000
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Variance	0.0205		0.0067	
Standard Deviation	14.3%		8.2%	

Note that stocks have a higher expected return than bonds and higher risk. Let us turn now to a portfolio with 50% invested in bonds and 50% invested in stocks.

Quick question: What is the sharpe ratio for the stock and bond fund, respectively?

$SR_1 = \frac{11\%}{14.3\%}$ $SR_2 = \frac{7\%}{8.2\%}$

Portfolio return

	Rate of Return			
Scenario	***Stock fund***	***Bond fund***	***Portfolio***	***squared deviation***
Recession	-7%	17%	5.0%	0.0016
Normal	12%	7%	9.5%	0.0000
Boom	28%	-3%	12.5%	0.0012
Expected return	11.00%	7.00%	9.0%	
Variance	0.0205	0.0067	0.0010	
Standard Deviation	14.31%	8.16%	3.08%	

The rate of return on the portfolio is a weighted average of the returns on the stocks and bonds in the portfolio:

$$r_P = w_B r_B + w_S r_S \qquad 5\% = 50\% \times (-7\%) + 50\% \times (17\%)$$

Portfolio return

	Rate of Return			
Scenario	*Stock fund*	*Bond fund*	*Portfolio*	*squared deviation*
Recession	-7%	17%	5.0%	0.0016
Normal	12%	7%	9.5%	0.0000
Boom	28%	-3%	12.5%	0.0012
Expected return	11.00%	7.00%	9.0%	
Variance	0.0205	0.0067	0.0010	
Standard Deviation	14.31%	8.16%	3.08%	

The *expected* rate of return on the portfolio is a weighted average of the *expected* returns on the securities in the portfolio.

$$E(r_P) = w_B E(r_B) + w_S E(r_S)$$

$$9\% = 50\% \times (11\%) + 50\% \times (7\%)$$

Portfolio variance

Scenario	Stock fund	Bond fund	Portfolio	squared deviation
	Rate of Return			
Recession	-7%	17%	5.0%	0.0016
Normal	12%	7%	9.5%	0.0000
Boom	28%	-3%	12.5%	0.0012
Expected return	11.00%	7.00%	9.0%	
Variance	0.0205	0.0067	0.0010	
Standard Deviation	14.31%	8.16%	3.08%	

The variance of the rate of return on the two risky assets portfolio is

$$\sigma_P^2 = (w_B\sigma_B)^2 + (w_S\sigma_S)^2 + 2w_B w_S \rho_{BS}\sigma_B\sigma_S$$

In our last example, we already have $\rho = -0.998$

Diversification effect

	Rate of Return			
Scenario	***Stock fund***	***Bond fund***	***Portfolio***	***squared deviation***
Recession	-7%	17%	5.0%	0.0016
Normal	12%	7%	9.5%	0.0000
Boom	28%	-3%	12.5%	0.0012
Expected return	11.00%	7.00%	9.0%	
Variance	0.0205	0.0067	0.0010	
Standard Deviation	14.31%	8.16%	3.08%	

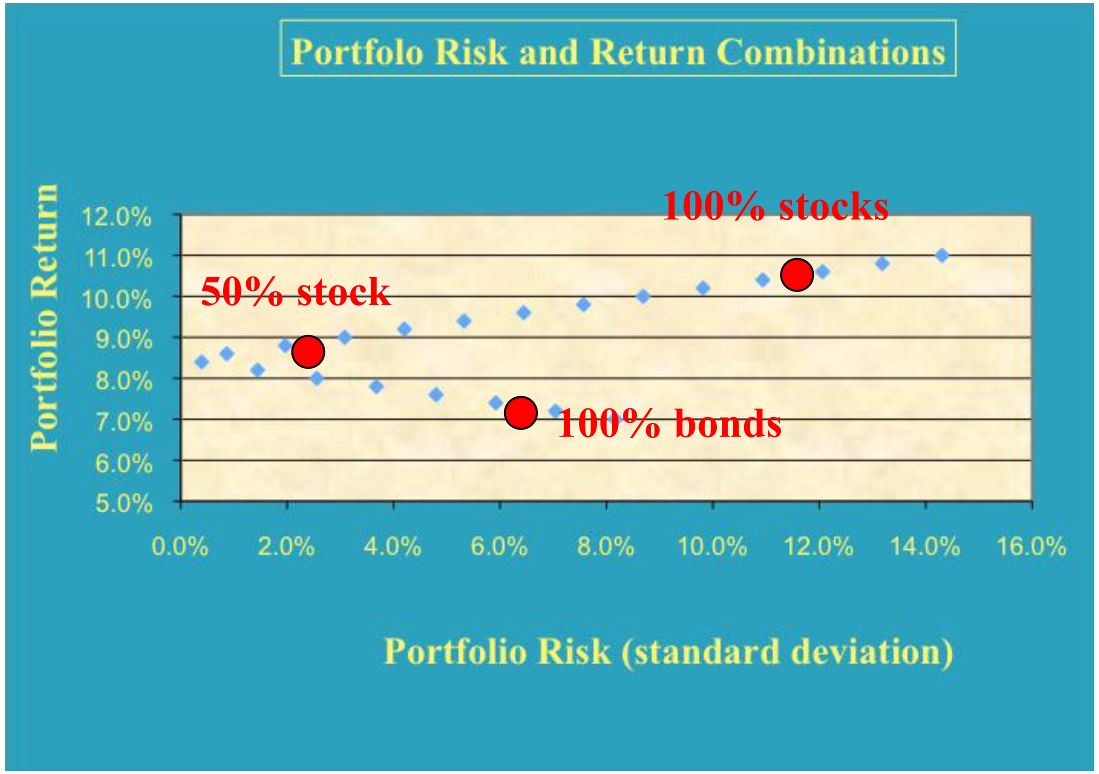
Diversification effect: An equally weighted portfolio (50% in stocks and 50% in bonds) has less risk than either stocks or bonds held in isolation.

Quick question: when ρ >0, is there a diversification effect? no

Opportunity set for two assets

机会集

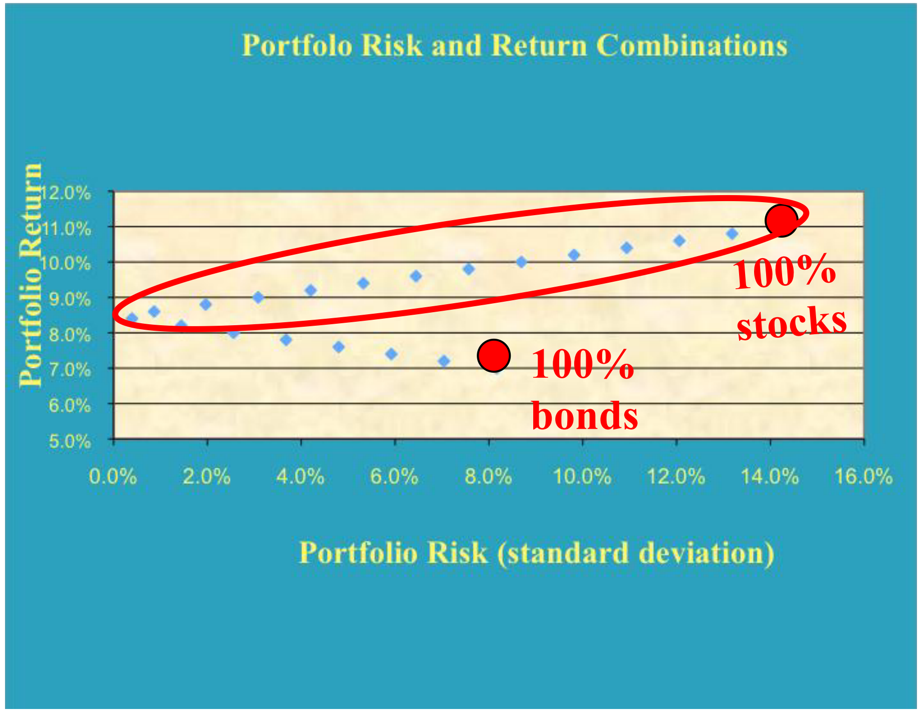
% in stocks	Risk	Return
0%	8.2%	7.0%
5%	7.0%	7.2%
10%	5.9%	7.4%
15%	4.8%	7.6%
20%	3.7%	7.8%
25%	2.6%	8.0%
30%	1.4%	8.2%
35%	0.4%	8.4%
40%	0.9%	8.6%
45%	2.0%	8.8%
50.00%	3.08%	9.00%
55%	4.2%	9.2%
60%	5.3%	9.4%
65%	6.4%	9.6%
70%	7.6%	9.8%
75%	8.7%	10.0%
80%	9.8%	10.2%
85%	10.9%	10.4%
90%	12.1%	10.6%
95%	13.2%	10.8%
100%	14.3%	11.0%



In above example, we only consider 50-50 combination. We can vary the weights and get the whole **opportunity set**. You cannot achieve other risk-return points outside the opportunity set.

Efficient frontier for two assets

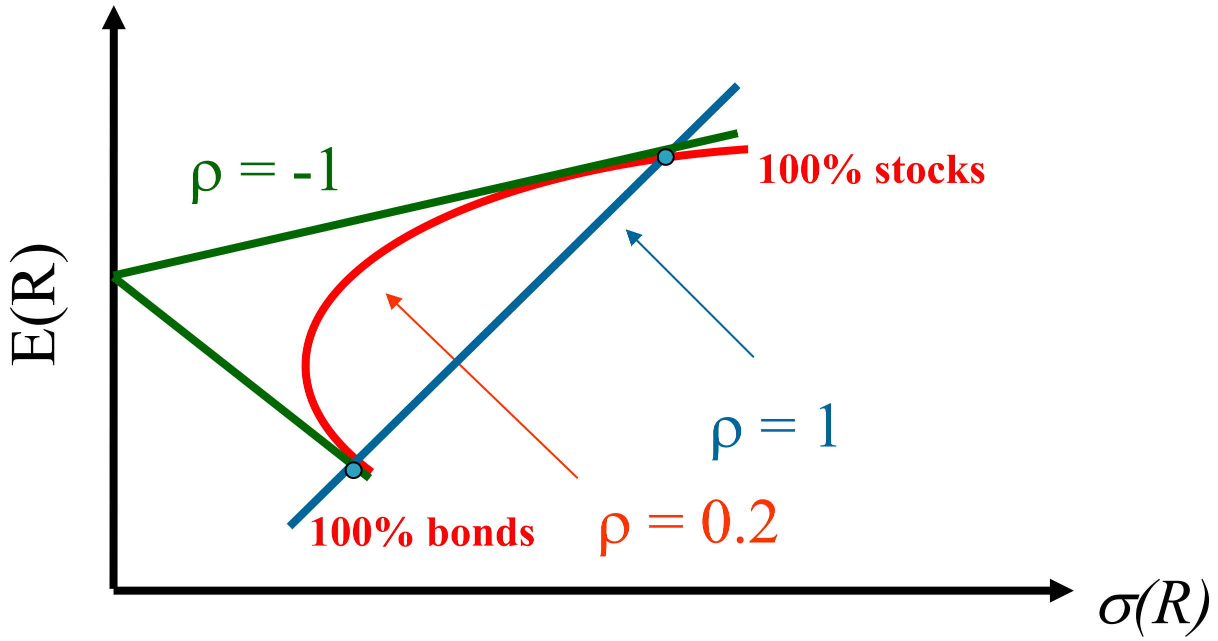
% in stocks	*Risk*	*Return*
0%	8.2%	7.0%
5%	7.0%	7.2%
10%	5.9%	7.4%
15%	4.8%	7.6%
20%	3.7%	7.8%
25%	2.6%	8.0%
30%	1.4%	8.2%
35%	0.4%	8.4%
40%	0.9%	8.6%
45%	2.0%	8.8%
50%	3.1%	9.0%
55%	4.2%	9.2%
60%	5.3%	9.4%
65%	6.4%	9.6%
70%	7.6%	9.8%
75%	8.7%	10.0%
80%	9.8%	10.2%
85%	10.9%	10.4%
90%	12.1%	10.6%
95%	13.2%	10.8%
100%	14.3%	11.0%



Among the opportunity set, the upper line are “better” than the lower one since they have higher returns for the same level of risk or less. It is the **efficient frontier**.

有效边界

How correlation affect the opportunity set?



$$w_B = 1 - w_S$$

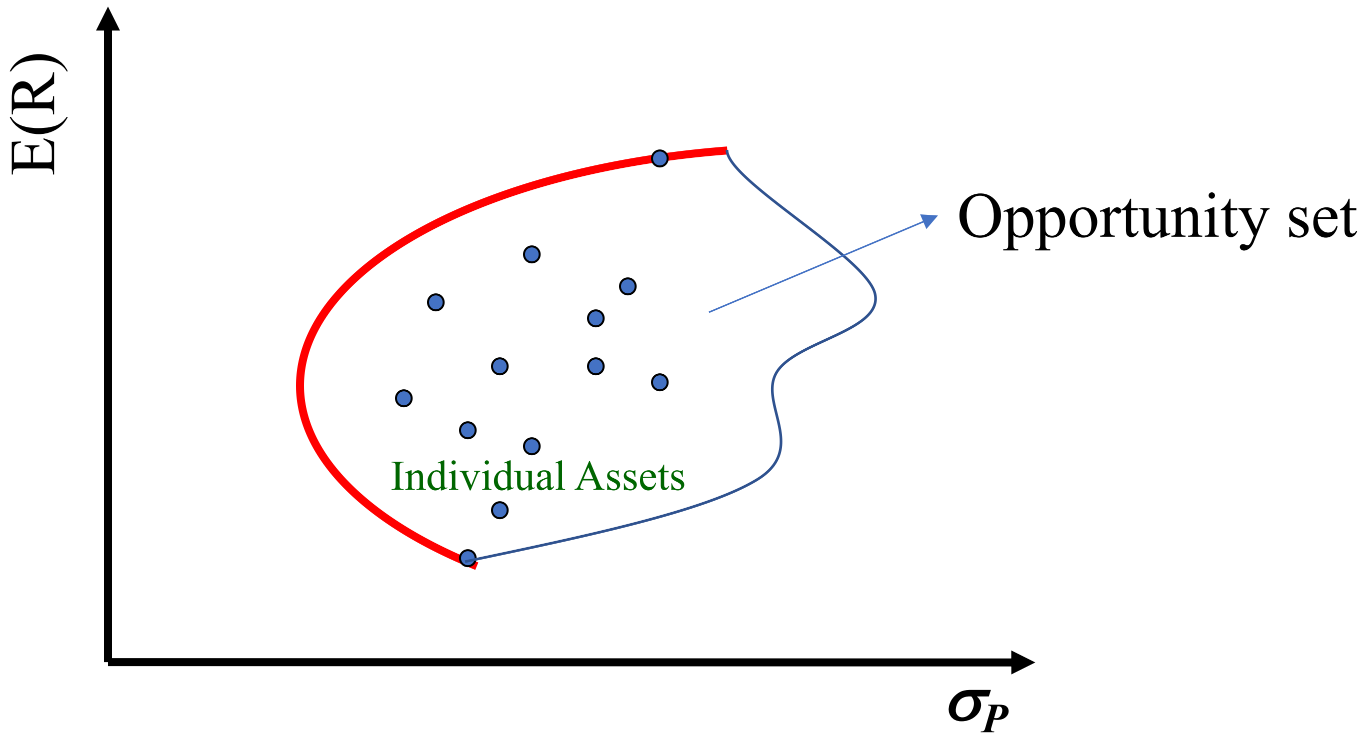
$$E(r_P) = w_B E(r_B) + w_S E(r_S)$$

$$\sigma_P^2 = (w_B\sigma_B)^2 + (w_S\sigma_S)^2 + 2w_B w_S \rho_{BS}\sigma_B\sigma_S$$

Opportunity set depends on correlation coefficient: $-1 \leq \rho \leq +1$

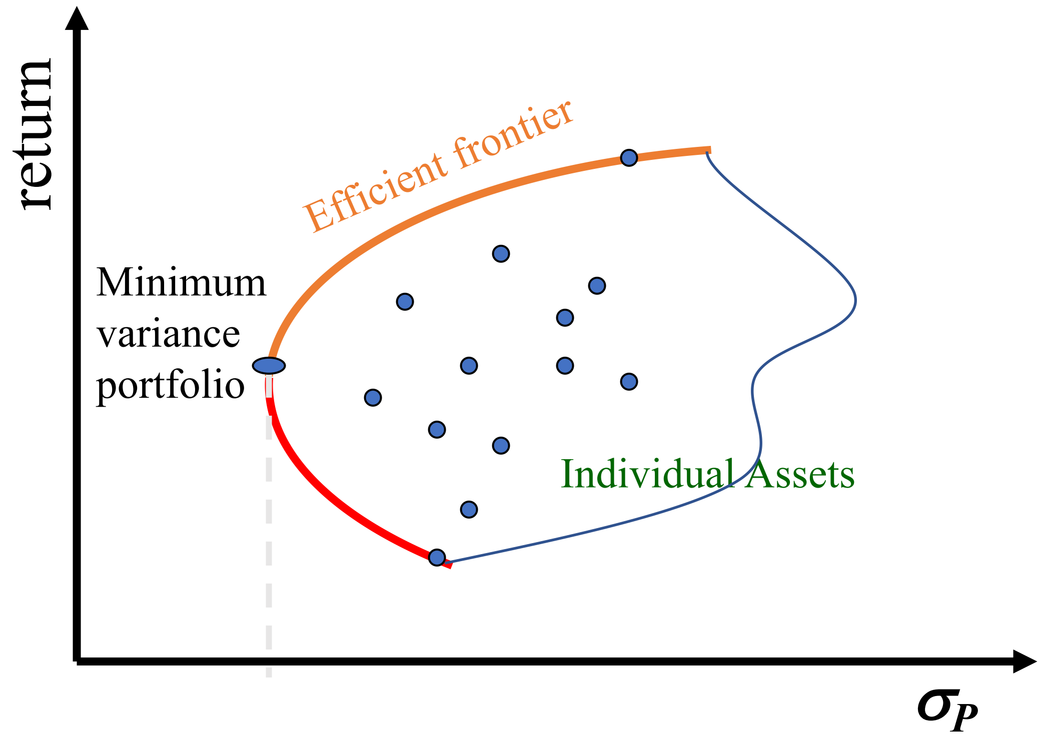
- If $\rho = +1$, no risk reduction is possible
- If $\rho = -1$, complete risk reduction is possible

Opportunity set for many assets



Consider a world with many risky assets; we can still identify the opportunity set of risk-return combinations of various portfolios.

Efficient frontier for many assets



- The question is: are there any points in the opportunity set dominate the others (high return but low risk)? The answer is the section of the opportunity set above the **minimum variance portfolio**.

- We call it the **efficient frontier**. It is part of opportunity set, which offers the highest return given the same risk or offers the lowest return given the same return.

Diversification and Portfolio Risk

多元化

- One advantage of portfolio investment is diversification. It can substantially reduce the variability of returns without an equivalent reduction in expected returns.

- when ρ is exactly 1, no diversification can happen. If not, diversification exists even when ρ>0.

- This risk reduction can arise because worse than expected returns from one asset are offset by better than expected returns from another. There is a minimum level of risk that cannot be diversified away, and that is the systematic portion.

Quick question: Which stock should you buy for the beginning cases?

Risk: Systematic vs. unsystematic

- A systematic risk is any risk that affects a large number of assets, each to a greater or lesser degree.
  - Examples: uncertainty about general economic conditions, such as GDP, interest rates or inflation; Policy uncertainty around presidential election; Covid-19; War;
  - Systematic risk cannot be diversified away since the risk arises from outside the “system” or the capital market. Recall the minimum variance portfolio. 外生的

- An unsystematic (or idiosyncratic) risk is a risk that specifically affects a single asset or small group of assets.
  - Announcements specific to a single company are examples of unsystematic risk.
  - Unsystematic risk can be diversified away via portfolio investment.

Jack Ma’ comment on systematic risk

25 Oct 2020, in the Bund Summit, Jack Ma said:

“中国金融没有系统性风险，因为中国的金融基本没有系统。”

Total risk

- Total risk = systematic risk + unsystematic risk
- The standard deviation of returns is a measure of **total risk**. We will talk about the measures for systematic risk and unsystematic in the next class.
- For well-diversified portfolios, unsystematic risk is very small. Consequently, the total risk for a diversified portfolio is essentially equivalent to the systematic risk.