Difference-in-Differences I: Basics

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Outline

1. Panel Data
2. Fixed Effect Estimation
3. Difference-in-Differences: Basics

Panel data

- A data set where we can observe the same units over more than one time period
- Pooled cross-section data
- Depending on the level of aggregation, the data demand can be quite different for the fixed effect estimation and the difference-in-differences estimation
- Which one is more data-demanding?
  - Individual fixed effect estimation ✓
  - Canonical two-way fixed effect estimation for the DID approach

DID要求更低.

APC Model

age = period - birth year

年龄 周期 出生状况. 完全共线性

- The effect of age, period, and cohort
- How to disentangling the effect of age, period, and cohort

Dohmen, T., A. Falk, B. Golsteyn, D. Huffman and U. Sunde (2017). 'Risk attitudes across the life course', *The Economic Journal*, vol. 127(605), pp. F95-F116.

Fitzenberger, B., G. Mena, J. Nimczik and U. Sunde (2021). 'Personality Traits Across the Life Cycle: Disentangling Age, Period and Cohort Effects*', *The Economic Journal*, vol. 132(646), pp. 2141-2172.

Outline

Fixed Effect Estimation

- **Setup of Fixed Effect Model**

$$y_{it} = x_{it}\beta + c_i + u_{it} \quad i = 1, 2, \ldots, N; t = 1, 2, \ldots, T$$

Where c_i, unobserved heterogeneity; u_{it}, the idiosyncratic error; $c_i + u_{it}$, composite error term

- **Fixed effect vs. random effect**
  - c_i is correlated with x_{it}, fixed effect
  - c_i is uncorrelated with x_{it}, random effect model
    - The most important consequence of random effects is that the residual for a given person are correlated across periods
- **Two-way fixed effect model**

$$y_{it} = x_{it}\beta + c_i + \lambda_t + u_{it} \quad i = 1, 2, \ldots, N; t = 1, 2, \ldots, T$$

Causal Framework of FE

- **Union membership vs. wage**
- Do workers whose wages set by collective bargaining earn more because of the union membership, or would they earn more anyway (perhaps because they are more experienced or skilled)
  - Y_{it}: the observed earnings of worker i at time t
  - Y_{0it} or Y_{1it} depending on the union status
  - D_{it}: the union status

Causal Framework of FE

- Suppose that (CIA is satisfied)

$$E\left(Y_{0it} \mid A_i, X_{it}, t, D_{it}\right) = E\left(Y_{0it} \mid A_i, X_{it}, t\right)$$

Where X_{it} is a vector of observed time-varying covariates and A_i is a vector of unobserved but fixed confounder, "Ability".
It means union status is as good as randomly assigned conditional on A_i and X_{it}

- Suppose that

$$E\left(Y_{0it} \mid A_i, X_{it}, t\right) = \alpha + \lambda_t + A_i'\gamma + X_{it}'\beta$$

Then (additive and constant causal effect)

$$E\left(Y_{1it} \mid A_i, X_{it}, t\right) = E\left(Y_{0it} \mid A_i, X_{it}, t\right) + \rho$$

Which then implies

Causal Framework of FE

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  Where X_{it} is a vector of observed time-varying covariates and A_i is a vector of unobserved but fixed confounder, "*Ability*". It means union status is as good as randomly assigned conditional on A_i and X_{it}
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- Then (additive and constant causal effect)

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  Which then implies

$$E\left(Y_{it} \mid A_i, X_{it}, t\right) = \alpha + \lambda_t + A_i'\gamma + X_{it}'\beta + \rho D_{it}$$

Causal Framework of FE

- Then, the empirical function will be

$$Y_{it} = \alpha_i + \lambda_t + \rho D_{it} + X_{it}'\beta + \varepsilon_{it}$$

where $\alpha_i \equiv \alpha + A_i'\gamma, \varepsilon_{it} \equiv Y_{0it} - E(Y_{0it}|A_i, X_{it}, t)$

Three ways to estimate

- **Method 1: Dummy Variable Estimation**
  - Treating the fixed effects, α_i, as parameters to be estimated; the year fixed effect, τ_t, is also treated as a parameter to be estimated.
  - The unobserved individual effects are coefficients on dummies for each individual, while the year effects are coefficients on time dummies.
- **Method 2: Within estimator, demeaning (deviation from means) estimator, or fixed effect estimator**
  - Method 1 and Method 2 are equivalent
- **Method 3: Differencing**

Demeaning regression: absorbing the fixed effect

- Fixed effects transformation or within transformation
  1. **Calculate the individual averages**

$$\overline{y}_i = \overline{\mathbf{x}}_i \boldsymbol{\beta} + c_i + \overline{u}_i \quad i = 1, 2, \ldots, N$$

Where $\overline{y}_i = \frac{1}{T}\sum_{t=1}^{T} y_{it}$, $\overline{\mathbf{x}}_i = \frac{1}{T}\sum_{t=1}^{T} \mathbf{x}_{it}$, $\overline{u}_i = \frac{1}{T}\sum_{t=1}^{T} u_{it}$

  2. **Make a demeaning**

$$y_{it} - \overline{y}_i = (\mathbf{x}_{it} - \overline{\mathbf{x}}_i)\boldsymbol{\beta} + (u_{it} - \overline{u}_i)$$

Or

$$\ddot{y}_{it} = \ddot{\mathbf{x}}_{it}\boldsymbol{\beta} + \ddot{u}_{it}$$

- Then the unobserved individual effect is killed

Differencing

- Make a difference between one year and the year before

$$y_{it} - y_{it-1} = (x_{it} - x_{it-1})\,\beta + (u_{it} - u_{it-1})$$

Or

$$\Delta y_{it} = \Delta x_{it}\beta + \Delta u_{it}$$

- With two periods, differencing is algebraically the same as demeaning, but not otherwise.
- With more than two periods, homoskedastic and serially uncorrelated u_{it}, demeaning is more efficient
- Differenced residuals are serially correlated.

Freeman (1984): Estimated effects of union status on wages

Estimated effects of union status on wages, Freeman (1984)

Survey	Cross Section Estimate	Fixed Effects Estimate
May CPS, 1974–75	.19	.09
National Longitudinal Survey of Young Men, 1970–78	.28	.19
Michigan PSID, 1970–79	.23	.14
QES, 1973–77	.14	.16

Notes: Adapted from Freeman (1984). The table reports cross section and panel (fixed effects) estimates of the union relative wage effect. The estimates were calculated using the surveys listed in the left-hand column. The cross section estimates include controls for demographic and human capital variables.

Other Examples

- Ashenfelter and Krueger (1994) and Ashenfelter and Rouse (1998) estimate the returns to schooling using samples of twins, controlling for family-fixed effects.
- Ashenfelter and Krueger (1994) use cross-sibling reports to construct instruments for schooling differences between twins.
- Li, H., P. W. Liu, J. Zhang and N. Ma (2007). *'Economic returns to communist party membership: Evidence from urban Chinese twins', The Economic Journal*, vol. 117(523), pp. 1504-1520.

Caveats of FE

- At a minimum, therefore, it's important to avoid overly strong claims when interpreting fixed effects estimates (It is never a bad advice for an applied econometrician in any case).
- xtreg, fe (be cautious with the reported number of observations)
  xtreg2, fe
- Time-invariant covariates will be demeaned or differenced
- Always keep in mind the level your analysis is conducted

$$y_{ipt} = x_{ipt}\beta + c_i + \delta_p + \lambda_t + u_{ipt} \quad i = 1, 2, \ldots, N; t = 1, 2, \ldots, T$$

(δ_p might be redundant) 多余的

$$y_{ipt} = x_{ipt}\beta + \delta_p + \lambda_t + u_{ipt} \quad i = 1, 2, \ldots, N; t = 1, 2, \ldots, T$$

(If the endogeneity is due to individual-level fixed effect, then endogeneity is still there)

Outline

Difference-in-differences (Intuition)

- The fixed effects strategy requires panel data, that is, repeated observations on the same individuals (or firms, or whatever the unit of observation might be).
- Often, however, the regressor of interest varies only at a more aggregate or group level, such as state or cohort. (when using pooled cross-sectional data, the comparability of group composition should be justified)
- For example, state policies regarding health care benefits for pregnant workers may change over time but are fixed across workers within states. The source of OVB when evaluating these policies must, therefore, be unobserved variables at the state and year level.
- In some cases, group-level omitted variables can be captured by group-level fixed effects, an approach that leads to the **differences-in-differences (DD)** identification strategy.

The DD Pioneer John Snow (1855)

- John Snow (1855) studied cholera epidemics in London in the mid-nineteenth century
- Establish that cholera is transmitted by contaminated drinking water (as opposed to "bad air," the prevailing theory at the time).
- Most medical opinion about cholera transmission at that time was *miasma*, which said diseases were spread by microscopic poisonous particles that infected people by floating through the air. These particles were thought to be inanimate, and because microscopes at that time had incredibly poor resolution, it would be years before microorganisms would be seen.
- Treatments, therefore, tended to be designed to stop poisonous dirt from spreading through the air. But tried and true methods like quarantining the sick were strangely ineffective at slowing down this plague.
- Imagine, if you were in Snow's shoes, what will and can you do to verify your guess?

How to find truth: Compare to What

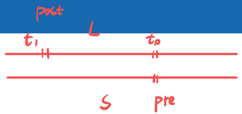


Table 70. Compared to what? Different companies.

Company	Outcome
Lambeth	$Y = L + D$
Southwark and Vauxhall	$Y = SV$

Table 71. Compared to what? Before and after.

Company	Time	Outcome
Lambeth	Before	$Y = L$
	After	$Y = L + (T + D)$

Table 72. Compared to what? Difference in each company's differences.

Companies	**Time**	**Outcome**	D_1	D_2
Lambeth	Before	$Y = L$		
	After	$Y = L + T + D$	$T + D$	
				D
Southwark and Vauxhall	Before	$Y = SV$		
	After	$Y = SV + T$	T	

Table 69. Modified Table XII (Snow 1854).

Company name	**1849**	**1854**
Southwark and Vauxhall	135	147
Lambeth	85	19

The Parallel Trends Assumption

- We are assuming that there is no time-variant company-specific unobservables. Nothing unobserved in Lambeth households that is changing between these two periods that also determines cholera deaths. (For example, mask distribution in the residence zone of Lambeth)
- This is equivalent to assuming that T is the same for all units: **"The parallel trends assumption"**
- Actually, two factors will lead to the failure of the assumption
  - The trend is actually not common (group-specific trend)
  - There might be other time-varying confounders

DD: A formal framework

Card and Krueger (1994)

- On April 1, 1992, New Jersey raised the state minimum from \$4.25 to \$5.05. Card and Krueger collected data on employment at fast-food restaurants in New Jersey in February 1992 and again in November 1992. These restaurants (Burger King, Wendy's, and so on) are big minimum-wage employers.
- Card and Krueger also collected data from the same type of restaurants in eastern Pennsylvania, just across the Delaware River. The minimum wage in Pennsylvania stayed at \$4.25 throughout this period.
- They used their data set to compute differences-in-differences (DD) estimates of the effects of the New Jersey minimum wage increase. That is, they compared the February-to-November change in employment in New Jersey to the change in employment in Pennsylvania over the same period.


宾汉顿
波基普西
康
斯克兰顿
扬斯敦
纽约
宾夕法尼亚州
印第安纳
阿伦敦
匹兹堡
哈里斯堡
新泽西州
威明顿
柏林
坎伯兰
黑格斯敦
温彻斯特
马里兰州
华盛顿
弗吉尼亚州
哈里森堡
钦科蒂格
沃彻普里格
里士满


兰斯代尔
Lansdale
纽敦
Newtown
特伦顿
Trenton
下南安普敦
Lower Southampton
西康舍霍肯
West Conshohocken
肯顿县
Kenton County
特拉华县
Delaware County
东格林尼治
East Greenwich
柏林
Berlin
威明顿
Wilmington

A Formal DD Setup: Potential Outcomes Framework

- Y_{1ist}: the potential fast food employment at restaurant i in state s and period t if there is a high state minimum wage
- Y_{0ist}: the potential fast food employment at restaurant i in state s and period t if there is a low state minimum wage (counterfactual outcomes)
- The heart of the DD setup is an additive structure for potential outcomes in the no-treatment state. Specifically, we assume that:

$$E(Y_{0ist} \mid s, t) = \gamma_s + \lambda_t$$

  where s denotes state (New Jersey or Pennsylvania) and t denotes period (February, before the minimum wage increase, or November, after the increase).
- This says that in the absence of a minimum wage change employment is given by a state effect and a time effect, which is assumed to be same in both states ("common trend hypothesis")

DD Setup: Potential Outcomes Framework

- Let D_{st} be a dummy for high-minimum-wage states and periods, assume that,

$$E(Y_{1ist} - Y_{0ist} \mid s, t) = \delta$$

- Then the observed employment in restaurant i as

$$Y_{ist} = \gamma_s + \lambda_t + \delta D_{st} + \varepsilon_{ist}$$

where $E(\varepsilon_{ist} \mid s, t) = 0$

DD Setup: Potential Outcomes Framework

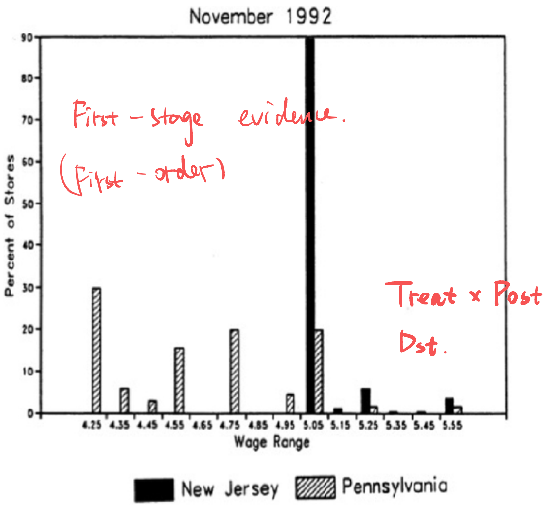
- From here, we can get

$$
\begin{aligned}
&E(Y_{ist} \mid s = NJ, t = Nov) - E(Y_{ist} \mid s = NJ, t = Feb) \\
&\qquad = (\gamma_{NJ} + \lambda_{Nov} + \delta) - (\gamma_{NJ} + \lambda_{Feb}) = \lambda_{Nov} - \lambda_{Feb} + \delta
\end{aligned}
$$

$$
\begin{aligned}
&E(Y_{ist} \mid s = PA, t = Nov) - E(Y_{ist} \mid s = PA, t = Feb) \\
&\qquad = (\gamma_{PA} + \lambda_{Nov}) - (\gamma_{PA} + \lambda_{Feb}) = \lambda_{Nov} - \lambda_{Feb}
\end{aligned}
$$

- The population difference-in-differences is the causal effect of interest.

$$
\begin{aligned}
&\{E(Y_{ist} \mid s = NJ, t = Nov) - E(Y_{ist} \mid s = NJ, t = Feb)\} \\
&- \{E(Y_{ist} \mid s = PA, t = Nov) - E(Y_{ist} \mid s = PA, t = Feb)\} = \delta
\end{aligned}
$$



- Figure 54. Distribution of wages for NJ and PA in November 1992.
- Notice how effective this is at convincing the reader that the minimum wage in New Jersey was binding. This piece of data visualization is not a trivial, or even optional, strategy to be taken in studies such as this.
- Beautiful pictures displaying the "first stage" effect of the intervention on the treatment are crucial in the rhetoric of causal inference

Average employment in fast food restaurants before and after the New Jersey minimum wage increase

Variable	PA (i)	NJ (ii)	Difference, NJ – PA (iii)
1. FTE employment before, all available observations	23.33 (1.35)	20.44 (.51)	−2.89 (1.44)
2. FTE employment after, all available observations	21.17 (.94)	21.03 (.52)	−.14 (1.07)
3. Change in mean FTE employment	−2.16 (1.25)	.59 (.54)	2.76 (1.36)

Notes: Adapted from Card and Krueger (1994), table 3. The table reports average full-time-equivalent (FTE) employment at restaurants in Pennsylvania and New Jersey before and after a minimum wage increase in New Jersey. The sample consists of all restaurants with data on employment. Employment at six closed restaurants is set to zero. Employment at four temporarily closed restaurants is treated as missing. Standard errors are reported in parentheses.

Identification assumption

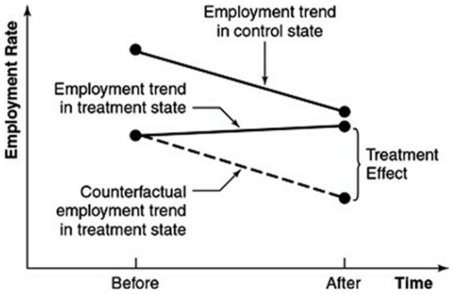
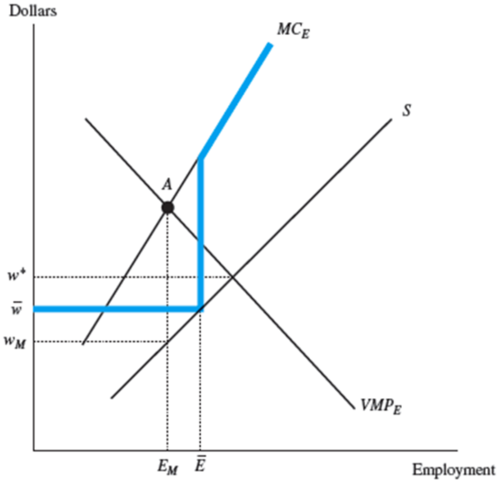


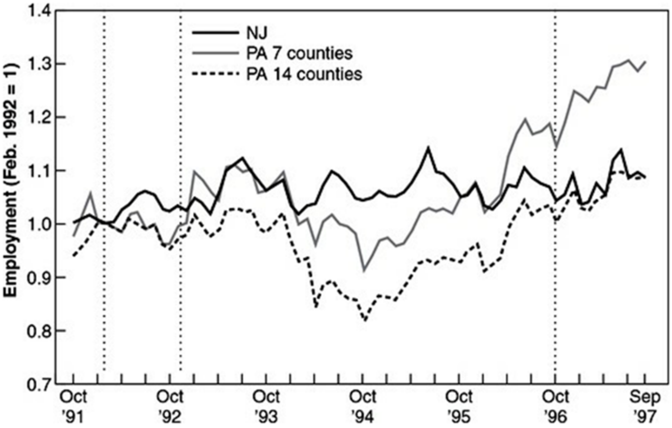
Figure: Causal effects in the DD model

- The key identifying assumption: employment trend would be same in both states in the absence of treatment.
- Treatment induces a deviation from this common trend, as illustrated in the figure.
- Although the treatment and control states can differ, this difference is meant to be captured by the state fixed effect, which plays the same role as the unobserved individual effect.

FIGURE 4-20 **The Impact of the Minimum Wage on a Nondiscriminating Monopsonist**

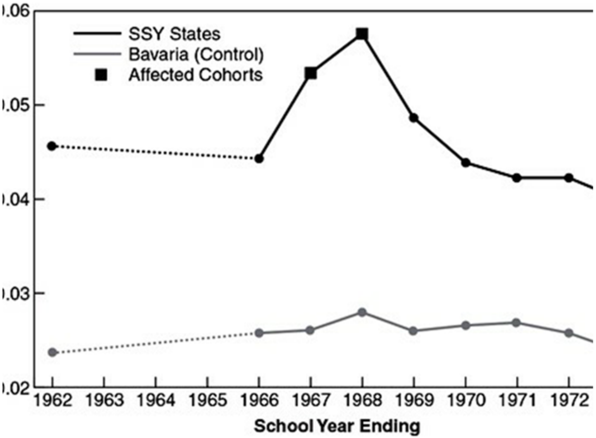
The minimum wage may increase both wages and employment when imposed on a monopsonist. A minimum wage set at $\overline{w}$ increases employment to $\overline{E}$.





However, to check out the pre-treat trend is neither necessary nor sufficient for the parallel trend hypothesis.

- The common trend assumption can be investigated using data on multiple periods: To plot the data is always a good idea.
- A poor example: (David Card and Alan B Krueger, 1994)



- A good example: Average grade repetition rates in second grade for treatment and control schools in Germany (from Pischke, 2007)
- The data span a period before and after a change in term length for students outside Bavaria (SSY states).

Regression DD (2 * 2)

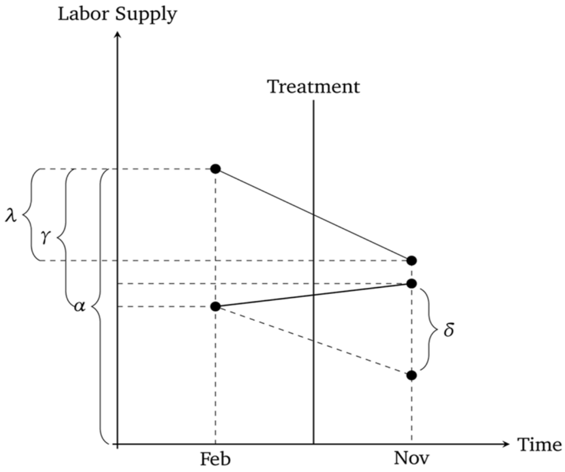
- **Regression function:**

$$y_{ist} = \alpha + \gamma NJ_s + \lambda d_t + \delta(NJ_s \times d_t) + \varepsilon_{ist}$$

- Let NJ_s be a dummy for restaurants in New Jersey and d_t be a time-dummy that switches on for observations obtained in November. Actually, we can write this in a more general and equivalent form as $D_{st} = NJ_s \times d_t$.

$$\begin{aligned}
\alpha &= E(Y_{ist} \mid s = PA, t = Feb) = \gamma_{PA} + \lambda_{Feb} \\
\gamma &= E(Y_{ist} \mid s = NJ, t = Feb) - E(Y_{ist} \mid s = PA, t = Feb) = \gamma_{NJ} - \gamma_{PA} \\
\lambda &= E(Y_{ist} \mid s = PA, t = Nov) - E(Y_{ist} \mid s = PA, t = Feb) \\
&= (\gamma_{PA} + \lambda_{Nov}) - (\gamma_{PA} + \lambda_{Feb}) = \lambda_{Nov} - \lambda_{Feb} \\
\delta &= \{E(Y_{ist} \mid s = NJ, t = Nov) - E(Y_{ist} \mid s = NJ, t = Feb)\} \\
&\quad - \{E(Y_{ist} \mid s = PA, t = Nov) - E(Y_{ist} \mid s = PA, t = Feb)\}
\end{aligned}$$

DD regression diagram



- Once again, the parallel trend hypothesis is the most crucial thing in a DD setup.

Thanks for your Attention!

Any questions or comments please write to:

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