Lecture 7 Real Business Cycle

25–41

不考 steady state (30–32)

Real Business Cycle

References:

King, R. G., & Rebelo, S. T. (1999). Resuscitating real business cycles. Handbook of macroeconomics, 1, 927-1007.
Romer, D (2018). Advanced macroeconomics.McGraw-Hill.
Galí, J. (2015). Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications. Princeton University Press.
McCandless, G. (2008). The ABCs of RBCs: an introduction to dynamic macroeconomic models. Harvard University Press.

Reading:

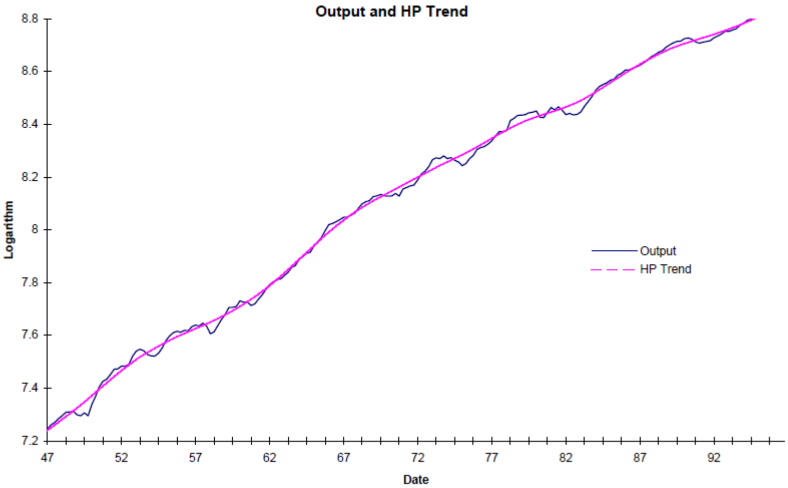
Romer Chapter 5
King & Rebelo, (1999)
Debate between Summers and Prescott (FED Minneapolis Quarterly Review Fall 1986)

Real Business Cycle

The RBC approach

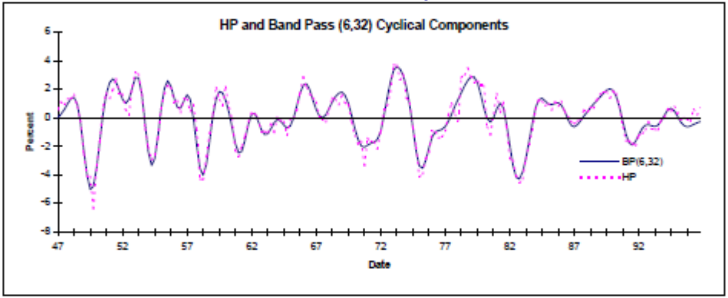
- Can we produce a unified account of business cycles and growth?
- Can an economic model with no imperfections, no monetary factors and no rationale for macroeconomic policy still explain the business cycle?
- Real factors can, in fact, drive the business cycle and explain long-run growth.

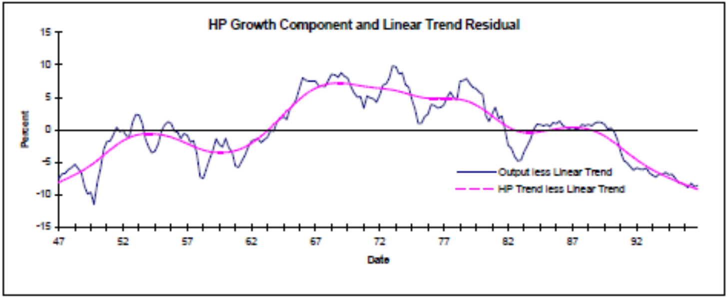
Trend and Fluctuations in Output



Source: King and Rebelo (1999)

Trend and Fluctuations in Output





Source: King and Rebelo (1999)

Business Cycle Facts

自回归

$y_i = \beta x_i + \varepsilon_i$

$y_t = \beta x_t + \varepsilon_t$ $\quad y_t = \rho y_{t-1} + \varepsilon_t$

Business Cycle Statistics for the U.S. Economy

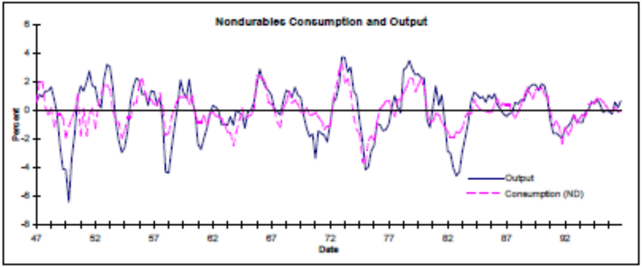
一阶自回归

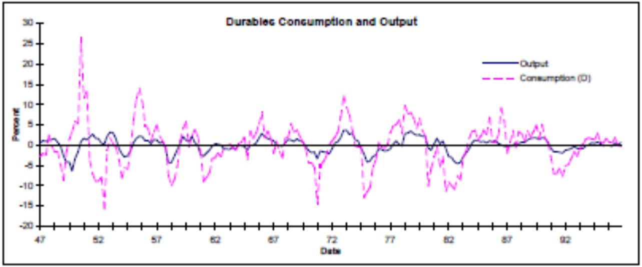
	Standard Deviation 波动性	Relative Standard Deviation	First Order Auto-correlation	Contemporaneous Correlation with Output
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30 +++	2.93	0.87	0.80
N	1.79	0.99	0.88	0.88
Y/N	1.02	0.56	0.74	0.55
w	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	-0.35
A	0.98	0.54	0.74	0.78

同向？ 正：顺周期 负：逆周期

Source: King and Rebelo (1999)。 All variables are in logarithms (with the exception of the real interest rate) and have been detrended with HP filter.

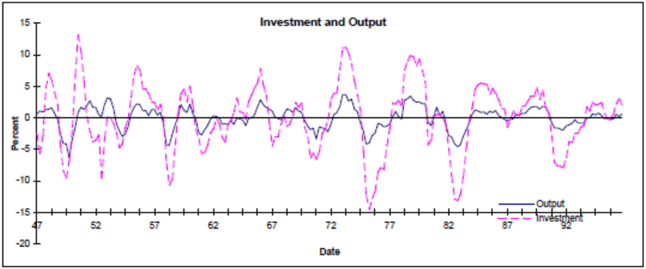
Consumption

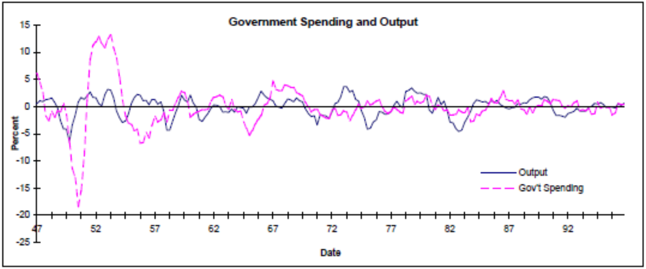




Source: King and Rebelo (1999)

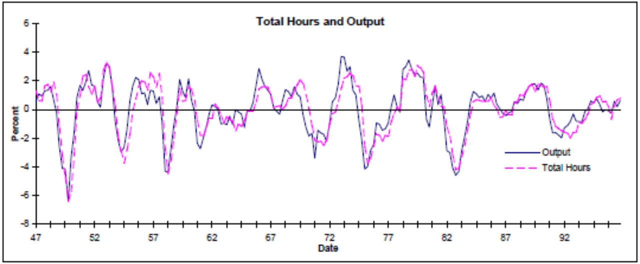
Investment and Government Spending

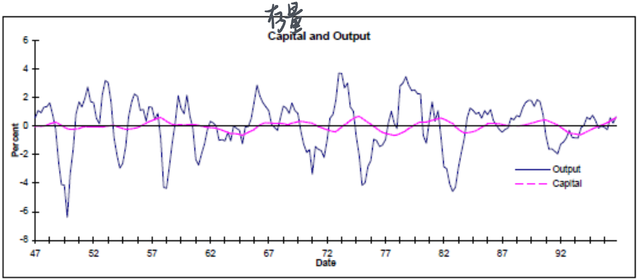




Source: King and Rebelo (1999)

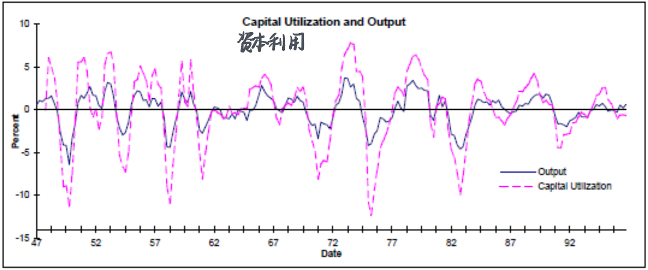
Total Hours and Capital

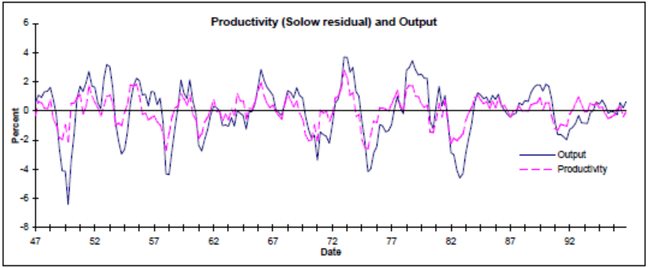




Source: King and Rebelo (1999)

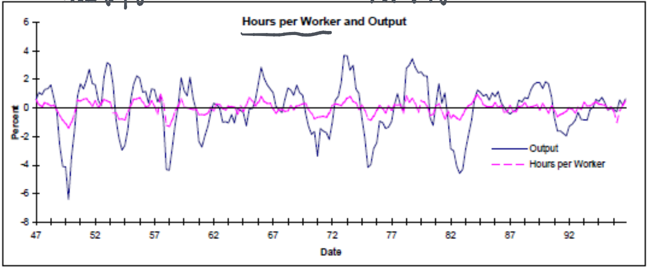
Utilization and Productivity

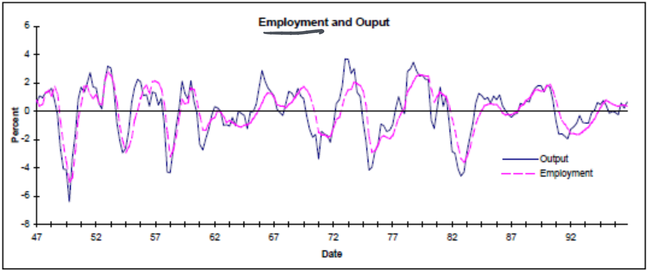




Source: King and Rebelo (1999)

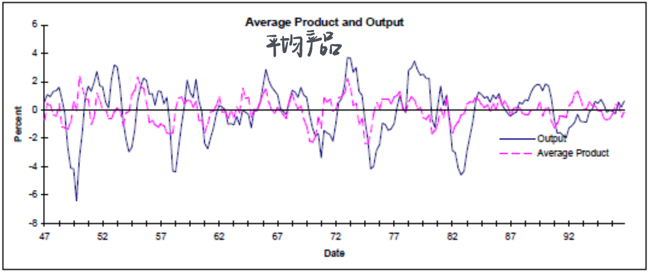
Intensive Margin and Extensive Margin

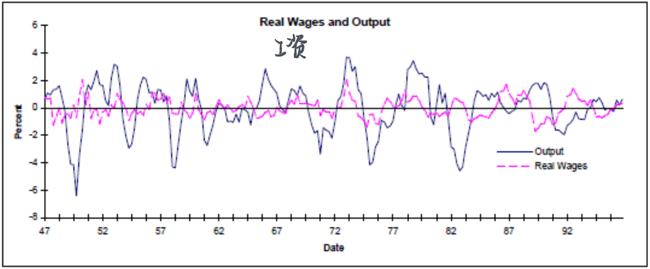




Source: King and Rebelo (1999)

Labor Productivity and Wages





工资具有刚性

Source: King and Rebelo (1999)

Implications of Stylized Facts

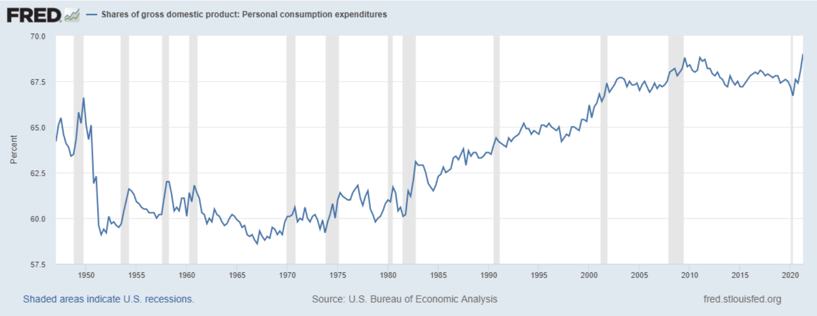
投资的高度不稳定性

- High volatility of investment $\rightarrow$ Keynes' assertion that investors have "animal spirits".

  主张

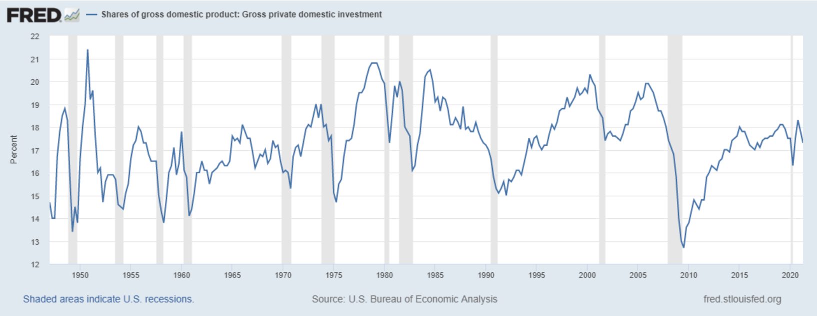
- Low cyclical volatility in capital implies that one can safely abstract from movements in capital in constructing a theory of economics fluctuations.
- High correlation between output and hours worked suggests importance of the labor market. But hours per worker, labor productivity and real wage are less volatile.
- The relative small variation in real wages suggests wage may not be an important allocative signal in the business cycle.

Consumption share



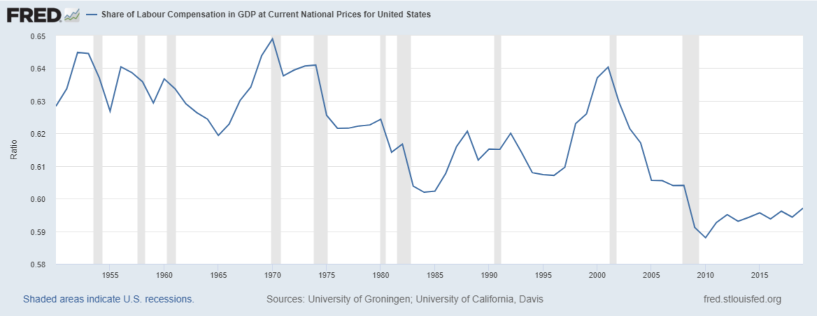
Source: FRED

Investment share



Source: FRED

The Labor share



Source: FRED

The Great Ratios and long-run trends

- GDP, consumption, investment all grow steadily over time
- C/Y and I/Y do not trend–common trend in most real aggregates. Impliess permanent level effects affect all series in the same way.
- Factor shares are relatively constant over time, or at least stationary
- Real wages have grown a lot. Hours per person have not.

Cycles and Trends for China

Several papers by Kaiji Chen, Tao Zha and their coauthors.

Real Business Cycle Outline

- Model setup and solutions
  - Decentralized equilibirum (versus Social planner's problem setup) 分散化决策
  - Find the steady state
  - Log linearization
- Calibrated RBC: successes and failures
  校准

The Basic Neoclassical Model

Preferences:

$$\sum_{t=0}^{\infty} b^t [U(C_t, L_t)], 0 < b$$

Where $U_c > 0$,$U_{cc} < 0$,$U_l > 0$,$U_{ll} < 0$. Infinite horizon is an approximation.

l是闲暇.

Endowments:

工作时间 闲暇时间

$$N_t + L_t = 1$$

Technolgy:

波动 增长

$$Y_t = A_t F(K_t, N_t X_t)$$

of which X_t is the deterministic component of productivity. and

$$X_{t+1} = \gamma X_t, \gamma > 1$$

恒定增速：$\gamma - 1$

The Basic Neoclassical Model

The output is used for consumption and investment:

$$Y_t = C_t + I_t$$

The capital stock evolves:

$$K_{t+1} = I_t + (1 - \delta)K_t$$

where δ is the depreciation rate.
Initial conditions: $K_0 > 0$,$X_0 > 0$ and $A_0 > 0$.

The Basic Neoclassical Model

Rescale the variables by X to get rid of the steady state growth.
e.g $y_t = \frac{Y_t}{X_t}$. per efficient labor
Transformed utility function:

b→β 存在一定关系

$$\sum_{t=0}^{\infty} \beta^t [U(c_t, L_t)]$$

Constraints:

$$N_t = 1 - L_t$$
$$y_t = A_t F(k_t, N_t)$$
$$y_t = c_t + i_t$$
$$\gamma k_{t+1} = i_t + (1-\delta) k_t$$

$\frac{K_{t+1}}{\frac{X_{t+1}}{\gamma}} = I_t + (1-\delta)\frac{K_t}{X_t}$

Given this close correspondence, RBC analyses sometimes simply start with the transformed economy, omitting growth all together.

忽略

Restrictions: technology

- Production technology
  - Observations to be matched: constant factor shares and a balanced growth path.
  - For a balanced growth path to be feasible, technology must be labor-augmenting.

$$Y_t = A_t F(K_t, N_t X_t)$$

  - Transformed form:

$$y_t = A_t k_t^{\alpha} N_t^{1-\alpha}$$

- TFP process (外生给定)

$$log A_t = \rho \, log A_{t-1} + \epsilon_t$$

Restrictions: preferences

- Observations: preferences need to be consistent with long-run growth in macro aggregates, but no trend in hours per worker.
- King, Plosser and Rebelo (1988)

CRRA形式的.

$$\begin{cases} \frac{1}{1-\sigma}([C_t v(L_t)]^{1-\sigma} - 1) & \sigma \neq 1 \\ log\ C_t + log\ v(L_t) & \sigma = 1 \end{cases}$$

- De-trended: $\beta = b(\gamma)^{1-\sigma}$ and $X_0 = 1$:

$$E \sum_{t=0}^{\infty} \beta^t [U(c_t, L_t)]$$

- We'll use $v(L) = exp\frac{\theta}{1-\eta}(L_t^{1-\eta} - 1)$:

$$u(c_t, L_t) = log(c_t) + \frac{\theta}{1-\eta}(L_t^{1-\eta} - 1)$$

Decentralized Equilibrium: Households

$$E\sum_{t=0}^{\infty}\beta^t[log(c_t)+\frac{\theta}{1-\eta}(L_t^{1-\eta}-1)]$$

Budget Constraint:

利润

$$c_t+\gamma k_{t+1}-(1-\delta)k_t+\gamma b_{t+1}=r_t b_t+w_t(1-L_t)+r_t^k k_t+\Pi_t$$

$(r_t>1)$ 还本付息　工资　资本

$$C_t+\underbrace{K_{t+1}-(1-\delta)K_t}_{I_t}+B_{t+1}=r_t B_t+W_t(1-L_t)X_t+r_t^k K_t+\Pi_t \triangleq \pi_t X_t$$

带到 t+1 期, t 期购入　(考虑效率)

- P_t is the price of output c_t, for now normalized to one.
- b_{t+1} is holdings of real bond bought at price 1 at time t and yielding r_{t+1}.
- r_t is the gross real interest rate between periods $t-1$ and t.
- w_t is the real wage, r_t^k is the real rental rate.

Decentralized Equilibrium: Households

Households hold capital.

$\tilde{\lambda}_t = \beta^t \lambda_t$ - discounted Lagrange乘子

λ_t - undiscounted

$$\mathcal{L} = E\{\sum_{t=0}^{\infty} \beta^t [log(c_t) + \frac{\theta}{1-\eta}(L_t^{1-\eta} - 1)]$$

$$+ \sum_{t=0}^{\infty} \beta^t \lambda_t [r_t b_t + w_t(1 - L_t) + r_t^k k_t + \Pi_t - c_t - \gamma b_{t+1}$$

可要可不要

$$- \gamma k_{t+1} + (1-\delta)k_t]\}$$

$\theta L_t^{-\eta} = \lambda_t w_t$

$\Rightarrow \log\theta - \eta \log L_t = \log\lambda_t + \log w_t$

$\Rightarrow \frac{d\log L_t}{d\log w_t} = -\frac{1}{\eta}$

$\frac{d\log N_t}{d\log w_t} = \frac{1}{\eta}$ 劳动供给弹性

$$\{c_t\} : \quad \frac{1}{c_t} = \lambda_t$$

leisure vs consumption

Intratemporal：当期 $\{L_t\} : \quad \theta L_t^{-\eta} - \lambda_t w_t = 0$

$$\{b_{t+1}\} : \quad E_t[-\beta^{t+1}\lambda_{t+1} r_{t+1} + \gamma \beta^t \lambda_t] = 0$$

bond market

Intertemporal：跨期

$\gamma = 1$, Euler Eq. $\quad E_t[\beta \frac{c_t}{c_{t+1}} r_{t+1}] = \gamma$

$$\{k_{t+1}\} : E_t[\beta^{t+1}\lambda_{t+1}(r_{t+1}^k + 1 - \delta) - \gamma\beta^t\lambda_t] = 0$$

capital.

$$E_t[\beta \frac{c_t}{c_{t+1}}(r_{t+1}^k + 1 - \delta)] = \gamma$$

Decentralized Equilibrium: Firms

工资 租金

$$\max_{N_t, k_t} \{A_t k_t^{\alpha} N_t^{1-\alpha} - w_t N_t - r_t^k k_t\}$$

FOCs:

$$\{N_t\}: \quad MPL = \boxed{w_t} = (1-\alpha)\frac{y_t}{N_t}$$

$$\{k_t\}: \quad MPK = \boxed{r_t^k} = \alpha A_t k_t^{\alpha-1} N_t^{1-\alpha}$$

WHAT'S NEXT STEP?

We will introduce some terminology, and then we calculate the steady state before we log-linearize these first order conditions around the steady state.

Terminology

- The non-stochastic steady state is defined as a situation in which all variables are constant and where the only source of uncertainty (which is the stochastic part of productivity) is held at its unconditional mean. 保持在其无条件下的均值、
- A variable is a realization of something that can change (either deterministically or stochastically). Endogenous variables are variables whose values are determined "inside" of a model. Exogenous variables are variables whose values are determined "outside" of a model.
- "State" and "control" variables: Exogenous variables are always state variable, but endogenous variables can be either controls or states. Loosely, **"control"** variables are variables whose values are chosen in a model and are free to "jump" in response to new information.
- **State variables** are variables whose values agents need to know to make decisions. These variables are either exogenous (a productivity term, government spending) or endogenous (capital shocks, stocks of assets, etc).

The steady state

$\log A_{t+1} = \rho \log A_t + \varepsilon_t \Rightarrow A_{t+1} = A_t^{\rho} \cdot e^{\varepsilon_t}$

We're going to linearize around the deterministic steady state $A = \bar{A} = 1$. 标准化

Firm

$\{N_t\}$: MPL = $w_t = (1-\alpha)\frac{y_t}{N_t}$

$\{k_t\}$: MPK = $r_t^k = \alpha A_t k_t^{\alpha-1} N_t^{1-\alpha}$

- From the FOC for capital: FOC

$$r^k = \alpha\left(\frac{k}{N}\right)^{\alpha-1} = \frac{\gamma}{\beta} - 1 + \delta$$

$$\Rightarrow \frac{k}{N} = \left(\frac{\alpha}{\frac{\gamma}{\beta} - 1 + \delta}\right)^{\frac{1}{1-\alpha}}$$

$\{c_t\}$: $\frac{1}{c_t} = \lambda_t$

vs consumption

当期 $\{L_t\}$: $\theta L_t^{-\eta} - \lambda_t w_t = 0$

$\{b_{t+1}\}$: bond market $E_t[-\beta^{t+1}\lambda_{t+1} r_{t+1} + \gamma\beta^t\lambda_t] = 0$

跨期

$\gamma = 1$, Euler Eq. $E_t[\beta \frac{c_t}{c_{t+1}} r_{t+1}] = \gamma$

$\{k_{t+1}\}$: $E_t[\beta^{t+1}\lambda_{t+1}(r_{t+1}^k + 1 - \delta) - \gamma\beta^t\lambda_t] = 0$

capital $E_t[\beta \frac{c_t}{c_{t+1}}(r_{t+1}^k + 1 - \delta)] = \gamma$

Steady State: $C_t = C_{t+1}$.

- From the wage equation

$$w = (1-\alpha)\left(\frac{k}{N}\right)^{\alpha}$$

$$w = (1-\alpha)\frac{y_t}{N_t} = (1-\alpha)\frac{k_t^{\alpha} N_t^{1-\alpha}}{N_t} = (1-\alpha)\left(\frac{k}{N}\right)^{\alpha}$$

The steady state

- From the capital accumulation equation:

$$i = (\gamma - 1 + \delta)k$$

- The resource constraint

$$(\frac{k}{N})^{\alpha} N = c + (\gamma - 1 + \delta)k \qquad (1)$$

$$\frac{c}{N} = (\frac{k}{N})^{\alpha} - (\gamma - 1 + \delta)\frac{k}{N} \qquad (2)$$

- From the intratemporal consumption leisure condition:

$$\theta(1 - N)^{-\eta} = \frac{1}{c}(1 - \alpha)(\frac{k}{N})^{\alpha} \qquad (3)$$

$$\frac{c}{N} = \frac{1 - \alpha}{\theta}\frac{1 - N}{N}(\frac{k}{N})^{\alpha} \qquad (4)$$

we consider a special case $\eta = 1$.

The steady state

$$\left(\frac{k}{N}\right)^\alpha - (\gamma-1+\delta)\frac{k}{N} = \frac{1-\alpha}{\theta}\frac{1-N}{N}\left(\frac{k}{N}\right)^\alpha$$

- Get N:

$$N = \frac{\frac{1-\alpha}{\theta}(\frac{k}{N})^\alpha}{\frac{1-\alpha+\theta}{\theta}(\frac{k}{N})^\alpha - (\gamma-1+\delta)\frac{k}{N}}$$

- Steady state investment

$$i = (\gamma-1+\delta)\frac{k}{N}N$$

- Steady state output

$$y = (\frac{k}{N})^\alpha N$$

- Steady state consumption

$$c = N((\frac{k}{N})^\alpha - (\gamma-1+\delta)\frac{k}{N})$$

Log Linearization

- Taylor Expansion of $f(x)$ around x^*

$$f(x) = f(x^*) + \frac{f'(x^*)}{1!}(x - x^*) + \frac{f''(x^*)}{2!}(x - x^*)^2 + \cdots$$

- For a sufficiently smooth function, the higher order derivatives will be small.

$$f(x) = f(x^*) + f'(x^*)(x - x^*)$$

- Log Linearization: log it first, Taylor expand it then.

Log Linearization

- Example

$$y_t = A_t k_t^{\alpha} N_t^{1-\alpha}$$

- Take logs:

$$lny_t = lnA_t + \alpha lnk_t + (1-\alpha)lnN_t$$

- Do the Taylor expansion around the steady state values: 一阶 Taylor

steady state: $lny^* = lnA^* + \alpha lnk^* + (1-\alpha)lnN^*$.

$$lny^* + \frac{1}{y^*}(y_t - y^*) = lnA^* + \frac{1}{A^*}(A_t - A^*) +$$

$$\alpha lnk^* + \alpha\frac{1}{k^*}(k_t - k^*) +$$

$$(1-\alpha)lnN^* + (1-\alpha)\frac{1}{N^*}(N_t - N^*)$$

x是 steady state时 x的值.

$\hat{x}_t = \frac{x_t - x}{x} \times 100\%$ 与 steady state 相比变动率.

- With hat indicating percentage deviation from the steady state values, we get

$$\hat{y}_t = \hat{A}_t + \alpha\hat{k}_t + (1-\alpha)\hat{N}_t$$

Linearization

$\ln\theta - \eta \ln L_t = -\ln c_t + \ln w_t \Rightarrow$

$\theta L_t^{-\eta} = \frac{1}{c_t} w_t \Rightarrow -\eta \hat{L}_t = -\hat{c}_t + \hat{w}_t$

- Households
  - Labor supply:

$$-\eta \hat{L}_t = -\hat{c}_t + \hat{w}_t$$

$\{c_t\}$: vs consumption $\quad \frac{1}{c_t} = \lambda_t$

当期 $\{L_t\}$: $\quad \theta L_t^{-\eta} - \lambda_t w_t = 0$

$\{b_{t+1}\}$: bond market $\quad E_t[-\beta^{t+1}\lambda_{t+1} r_{t+1} + \gamma\beta^t\lambda_t] = 0$

$\gamma=1$, Euler Eq. $\quad E_t[\beta \frac{c_t}{c_{t+1}} r_{t+1}] = \gamma$

  - Euler:

$$E_t(\hat{c_{t+1}} - \hat{r_{t+1}}) = \hat{c}_t$$

$\{k_{t+1}\}$: $E_t[\beta^{t+1}\lambda_{t+1}(r^k_{t+1} + 1 - \delta) - \gamma\beta^t\lambda_t] = 0$

capital $\quad E_t[\beta \frac{c_t}{c_{t+1}}(r^k_{t+1} + 1 - \delta)] = \gamma$

  - Interest rate (gross):

$$r\hat{r}_t = (r - 1 + \delta)\hat{r}_t{}^k$$

$r^k_{t+1} + 1 - \delta = r_{t+1}$

$dr^k_{t+1} = dr_{t+1} \quad r^k \hat{r}^k_{t+1} = r\hat{r}_{t+1}$

$r^k + 1 - \delta = r$

$\Rightarrow r^k = r - 1 + \delta$

$(r-1+\delta)\hat{r}^k_t = r\hat{r}_t$

$\hat{x}_t = \frac{x_t - x}{x}$

$\Rightarrow x\hat{x}_t = x_t - x$

$x\hat{x}_t = dx_t$

- Firms
  - Labor demand:

$$\hat{w}_t = \hat{A}_t + \alpha(\hat{k}_t - \hat{N}_t)$$

  - Capital demand:

$$\hat{r_t}^k = \hat{A}_t + (\alpha - 1)(\hat{k}_t - \hat{N}_t)$$

$\{N_t\}$: MPL $= \quad w_t = (1-\alpha)\frac{y_t}{N_t}$

$\{k_t\}$: MPK $= \quad r^k_t = \alpha A_t k_t^{\alpha-1} N_t^{1-\alpha}$

Linearization

- Technology
  - Production function:

$$\hat{y_t} = \hat{A}_t + \alpha\hat{k_t} + (1-\alpha)\hat{N}_t$$

  - Time constraint:

$$N\hat{N}_t = -L\hat{L_t}$$

  - Resource constraint:

$$\frac{c}{y}\hat{c_t} + \frac{i}{y}\hat{i_t} = \hat{y_t}$$

- Law of motion for the states
  - Capital:

$$\gamma k \hat{k}_{t+1} = (1-\delta)k\hat{k}_t + i\hat{i}_t$$

$$\gamma\hat{k_{t+1}} = (1-\delta)\hat{k_t} + \frac{i}{k}\hat{i_t}$$

  - Stochastic process for A:

$$\hat{A_t} = \rho\hat{A_{t-1}} + \epsilon_t$$

Summary: 2 states: k, A; 8 controls: c, w, r^k, N, L, y, i, r; 8 equations.

Model Solution

8个变量 ⇒ 2个状态

$$\begin{bmatrix} \hat{c}_t \\ \hat{N}_t \\ \hat{L}_t \\ \hat{y}_t \\ \hat{i}_t \\ \hat{w}_t \\ \hat{r}_t^k \\ \hat{r}_t \end{bmatrix} = F \begin{bmatrix} \hat{k}_t \\ \hat{A}_t \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} \hat{k}_{t+1} \\ \hat{A}_{t+1} \end{bmatrix} = P \begin{bmatrix} \hat{k}_t \\ \hat{A}_t \end{bmatrix} \tag{6}$$

状态之间的转换

Calibrating the model

校准

- The basic idea of calibration is to choose parameter values on the basis of micro-economic evidence and then to compare the model's predictions concerning the variance and covariances of various series with those in the data.
- Two advantages of calibration: brings information from micro research; avoid the difficulties in statistical inference and interpretation.
- Bad side: no measurement about how good the model is.

Calibrating model's parameters

- Discount factor $b = 0.984$: steady state real rate coincides with the average return to capital in the economy (S&P 500, 6.5%).
- The labor share is around $2/3$, $\alpha = 1/3$.
- Growth rate of technical change set to match the long-run trends in GDP growth: $\gamma = 1.004$.
- Depreciation rate $\delta = 0.025$, set to match the empirical K/Y ratio.
- Utility is logarithmic in consumption, this implies an Elasticity of intertemporal substitution of 1.
- $\eta = 1$, $\theta = 3.48$ is chosen to match steady state hours worked, approximately 20% of time available.
- The persistence of the A process can be estimated from the de-trended Solow residual in the data $\rho = 0.979$.

The parameters table

Parameters	Description	Value
b	Discount Factor	0.984
θ	Relative importance of leisure	3.48
σ	Risk aversion for consumption	1
η	Risk aversion for leisure	1
γ	Growth rate for labor productivity	1.004
α	Cobb-D production capital share	0.333
δ	Per quarter depreciation rate	0.025
ρ	Persistence of Tech shock	0.979
σ_ϵ	SD of Tech shock	0.0072

Calibrating the technology process

$$Y_t = A_t K_t^{\alpha}(X_t N_t)^{1-\alpha}$$
$$logSR_t = logY_t - \alpha K_t - (1-\alpha)N_t$$
$$= logA_t + (1-\alpha)logX_t$$

X_t is a deterministic trend. Can de-trend $logSR_t$ and then estimate an AR(1).

Business cycle moments from the model

Business Cycle Statistics for Basic RBC Model[35]

	Standard Deviation	Relative Standard Deviation	First Order Auto-correlation	Contemporaneous Correlation with Output
Y	1.39	1.00	0.72	1.00
C	0.61	0.44	0.79	0.94
I	4.09	2.95	0.71	0.99
N	0.67	0.48	0.71	0.97
Y/N	0.75	0.54	0.76	0.98
w	0.75	0.54	0.76	0.98
r	0.05	0.04	0.71	0.95
A	0.94	0.68	0.72	1.00

Note: All variables have been logged (with the exception of the real interest rate)and detrended with the HP filter.

Source: King and Rebelo (1999)

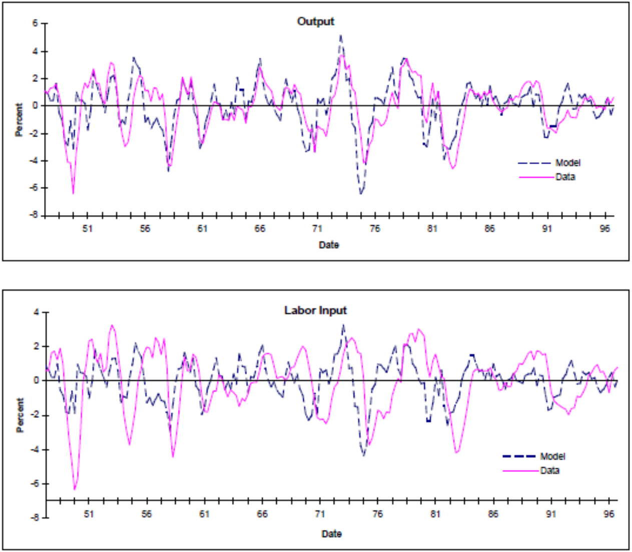
Data versus Model

		Data: Standard Deviation	Data: Relative Standard Deviation			Model: Standard Deviation	Model: Relative Standard Deviation
	Y	1.81	1.00	>	Y	1.39	1.00
	C	1.35	0.74	>>	C	0.61	0.44
good	I	5.30	2.93	>	I	4.09	2.95
	N	1.79	0.99	>>	N	0.67	0.48
	Y/N	1.02	0.56	>	Y/N	0.75	0.54
	w	0.68	0.38	<<	w	0.75	0.54
	r	0.30	0.16	>>	r	0.05	0.04
good	A	0.98	0.54	><	A	0.94	0.68

Data   Model

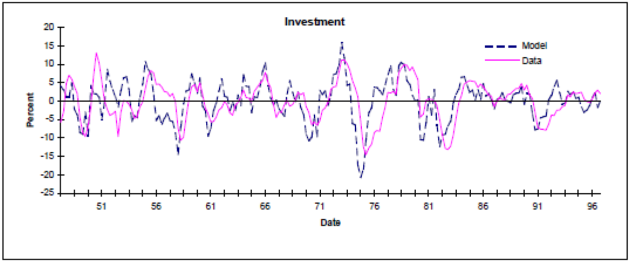
Source: King and Rebelo (1999)

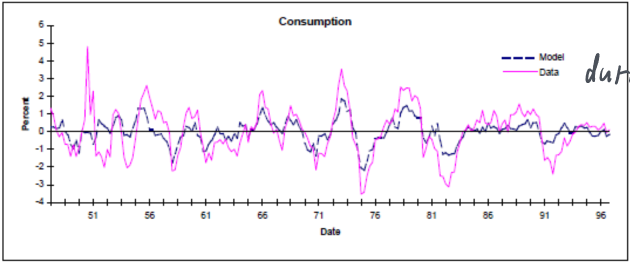
Model Simulations: Output and Labor Input



Source: King and Rebelo (1999)

Model Simulations: Investment and Consumption





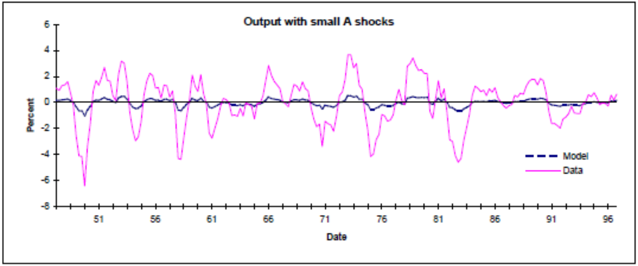
durable ?

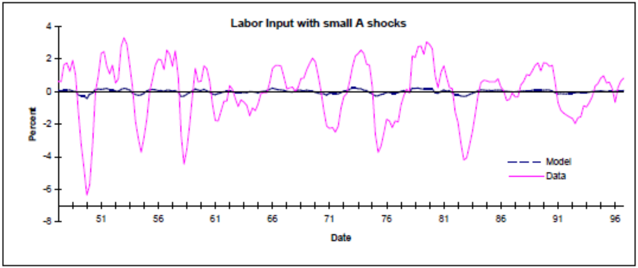
Source: King and Rebelo (1999)

Success and Criticisms

- Technology shocks are the dominant source of fluctuations. And Solow Residual has excessively large variation.
- Unreasonable degree of intertemporal substitution in labor supply.
- The model's strongly pro-cyclical real wage poses tension with empirical facts. (solved, wage smoothing, etc)
- Equity premium is incompatible with standard preferences.

Need Large Productivity Shocks





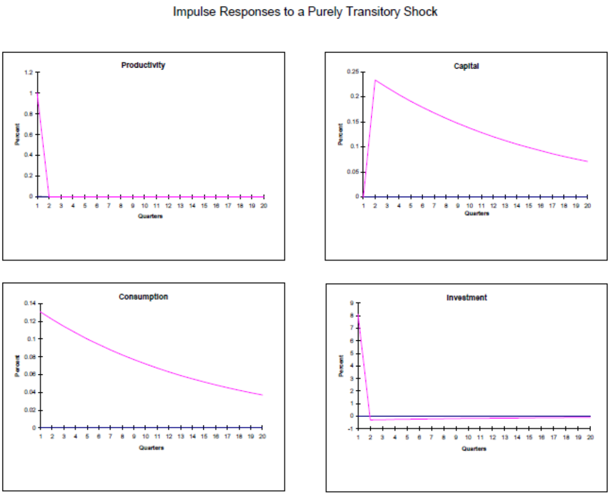
Source: King and Rebelo (1999)

Impulse Response Functions and Variance Decomposition

- Impulse Response Functions

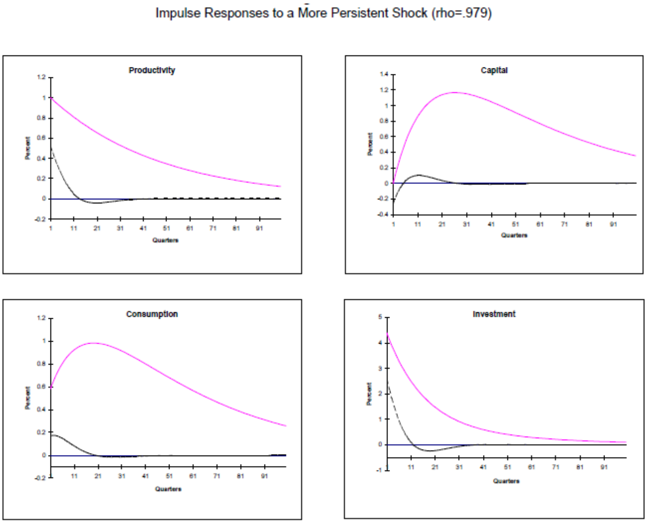
$$IRF(h) = \{E_t X_{t+h} - E_{t-1} X_{t+h}\} | \epsilon_t = e$$

Need Persistent Productivity Shocks



Source: King and Rebelo (1999)

Need Persistent Productivity Shocks



$(0.97)^{10} = 0.737$

Source: King and Rebelo (1999)

Inspecting the mechanism

- Production function in the first period

$$\hat{y}_t = \hat{A}_t + (1-\alpha)\hat{N}_t$$

- GDP response crucially depends on the labor response. A large y response, requires very elastic labor supply (RBC requires 2-4, microevidence of intensive elasticity is 0.5).
- We are richer: optimal to raise consumption today and in the future.
- There is smoothing: consumption goes up, but investment go up a lot.
- Permanent shock: wealth effects and substitution effects on labor supply cancel out.
- Temporary shock: wealth effects smaller, substitution effects larger.
- The MPL goes up, wage rises $\rightarrow$ substitution effect on labor supply.

Inspecting the mechanism: persistence

(*)

- As the persistence rises, the wealth effect becomes bigger. We'd like to consume and enjoy more leisure.
- All other things equal, the labor response is smaller.
- But $\cdots$ the leisure/consumption choice also depends on the interest rate.

Inspecting the mechanism: persistence

(*7)

- The persistence A profile induces dynamics in the real interest rate.
- Consider the FOCs for labor and the Euler equation. These imply:

$$\frac{1 - N_t}{1 - N_{t+1}} = \frac{\gamma}{\beta r_{t+1}} \frac{w_{t+1}}{w_t}$$

- The time-path for the real wage is relatively smooth.
- Interest rate more important for the dynamics of labor.
- Leisure costly when r higher, so still supply more labor while the interest rate is high.

Assessing the baseline RBC models

- Problems:
  - There is no real monetary effects; 货币
  - It relies on larger variation of tech shocks; 技术冲击
  - There is no deviation from non-Walrasian world 信贷不完全
- "real" extensions: indivisible labor (instensive and extensive margin of labor supply). Rogerson (1988) , Hansen (1985); multiple sectors and sector-specific shocks. Long and Plosser (1983). non-Walrasian: distortionary taxation.
- Capital utilization
- Remeasuring productivity

Extensions of the baseline model

- Technology and Non-Technology Shocks
  - Gali (1999) critique: Matching variances and covariances is a weak test when there may be multiple shocks.
    Model makes predictions for impulse responses to particular shocks, whic provides sharper test.
  - Uses structural VAR to decompose technology and non-technology shocks. ONLY tech shocks have permanent effect on productivity.
- Basu (2006) Tech improvements are contractionary in the short-run (totally inconsistent with RBC). Long-run effects are consistent with RBC model.

How about other shocks

Government spending shock will lead to countercyclical consumption.

Distortionary tax changes are not that frequent.

Money has small effects in Cash-in-advance models.

Non-neutrality of money

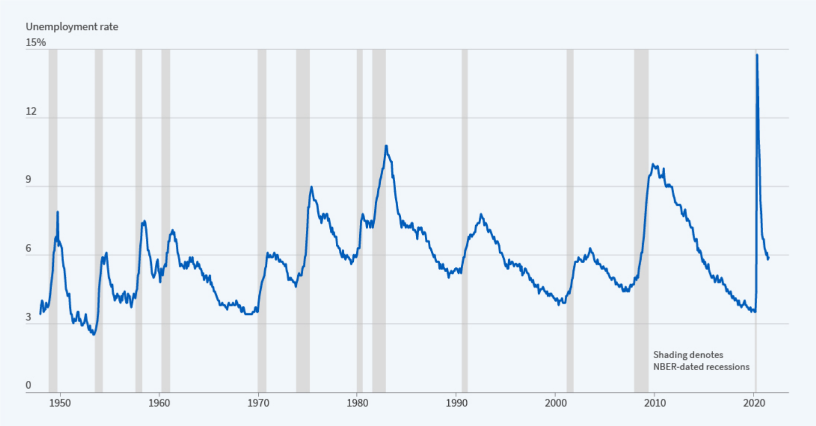
- Whether monetary changes have real effects. (Simple regression, Friedman and Schwartz (1963), Romer and Romer (1989), Cook and Hahn (1989), Kuttner (2001))
- VARs(Sims 1980) and Local Projections (Jorda 2005)

Final Remarks

Prescott (1986) :*"the match between theory and observation is excellent, but far from perfect."*
Plosser (1989): *"the whole idea that such a simple model with no government, no market failures of any kind, rational expectations, no adjustment costs could replicate actual experience this well is very surprising."*
Summers (1986): *"My view is that real business cycle models of the type urged on us by Prescott have nothing to do with the business cycle phenomena observed in the United States"*

APPENDIX

NBER recession



Source: NBER website

- Business cycle history: the pre-depression era; the great depression and World War II, the early post-war period, the great moderation, the great recession and its aftermath (and now the COVID-19).

A baseline RBC model

- Ramsey model + technology shock
- Difference between the current one with the Ramsey model: the inclusion of leisure in utility, and the shocks.
- An intertemporal equation links today's consumption and tomorrow's consumption
- An intratemporal equation links consumption and labor supply choice
- Assumptions to make the model analytically solvable: no government spending and fully depreciation of capital goods.

Social Planner's Problem and Decentralized Equilibrium

The two approaches are identical, because there are no market imperfections, so the first welfare theorem holds: the competitive, decentralized equilibrium is a solution to the planner problem.

Social Planner's Problem

Solve the following.

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c_t, L_t)$$
$$+ \sum_{t=0}^{\infty} \beta^t \lambda_t [A_t F(k_t, N_t) + (1-\delta) k_t - c_t - \gamma k_{t+1}]$$
$$+ \sum_{t=0}^{\infty} \beta^t \omega_t [1 - L_t - N_t]$$

FOCs:

$$c_t : u_1(c_t, L_t) = \lambda_t$$
$$L_t : u_2(c_t, L_t) = \omega_t$$
$$N_t : \lambda_t A_t F_2(k_t, N_t) = \omega_t$$
$$k_{t+1} : \beta \lambda_{t+1} [A_{t+1} F_1(k_{t+1}, N_{t+1}) + 1 - \delta)] = \gamma \lambda_t$$

here ω is not wage w.

Social Planner's Problem: Bellman Equation Approach

Write planner's problem as a Bellman equation:

$$V(A_t, k_t) = \max_{c_t, N_t, k_{t+1}} u(c_t, N_t) + \beta E_t V(A_{t+1}, k_{t+1})$$

$$s.t. A_t k_t^{\alpha} N_t^{1-\alpha} = c_t + k_{t+1}$$

Appendix

- Rational expectations says the expectations of future realizations of relevant variables are
  - correct on average
  - the forecast errors are unpredictable given available information
- In other words, agents have model consistent expectations in the sense that
  - agents know the model generating endogenous variables
  - and they use this knowledge to make forecasts.

Appendix

Lucas Critique (Lucas 1976)

- it is fraught with hazard to try to predict the effects of a policy change based on correlations (or regression coefficients) based on historical data.
- A parameter is structural if it is invariant to the rest of economic environment (policy environment).
- A parameter is reduced-form if it is not invariant to the environment.