New Keynesian Model

1-33 (HW3)

Outline

- Gali Chapter 2 (skipped)
  - Perfect competition, flexible prices and wages: monetary policy has effects only on nominal variables.
  - Even money in the utility, the resulting non-neutrality is quantitatively small and empirically little relevant.
- The Baseline New Keynesian Model 垄断竞争 价格粘性
- Early Critiques of the NK Model
- Medium-Scale NK Models
  - Credible Disinflation
  - Sticky Inflation and Responses
- Recent Critiques and Tests of the New Keynesian Model
  - The Minnesota Critique
  - The Cochrane Critique: Taylor Rule and Indeterminancy
  - The New Keynesian Phillips Curve in the Data.

Roadmap

- The textbook New Keynesian model includes:
  - Monopolistic Competition, product differentiation
  - Nominal Rigidity, staggered price setting (Calvo (1983))

The Three Blocks

Three "blocks" to the model and the three equations. (IS-LM?)

- Households and IS curve
- Firms and the Phillips curve
- Monetary authority and a monetary policy rule to close the model.

Investment - Saving $$\tilde{y}_t = -\frac{1}{\sigma} E_t \left\{ \hat{i_t} - \hat{\pi_{t+1}} - \hat{r_t^n} \right\} + E_t \{ \tilde{y_{t+1}} \}$$

Inflation - Output $$\hat{\pi}_t = \kappa \tilde{y}_t + \beta E_t \left\{ \hat{\pi_{t+1}} \right\}$$

Nominal Interest Rate $$\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_y \tilde{y}_t + v_t$$

$\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$

$\kappa = \lambda(\sigma + \varphi)$.

$\theta \uparrow$, $\lambda \downarrow$, $\kappa \downarrow$.

Phillips 曲线变平坦.

Households

Labor

需求冲击

$$\max_{C_t, N_t, B_t} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) Z_t$$

利润分红

$$s.t.\ \ C_t + Q_t \frac{B_t}{P_t} = \frac{B_{t-1}}{P_t} + \frac{W_t}{P_t} N_t + \frac{\Pi_t}{P_t}$$

FOCs:

$$\lambda_t = Z_t C_t^{-\sigma}$$

$$\lambda_t w_t = Z_t N_t^{\varphi}$$

名义量：

$$Q_t = 1/R_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}}$$

where $w_t = W_t / P_t$

Consumption Varieties

混合消费束

- C_t is the choice of the composite consumption bundle
- Structure of monopolistic competition follows Dixit and Stiglitz (1977). Different varieties of goods

ϵ: 可替代性

CES Aggregation:
$$C_t = \left[\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}$$

- ϵ is the price elasticity of demand for variety j

Maximization of C_t

$[\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}} dj]^{\frac{\epsilon}{\epsilon-1}}$

$\frac{\epsilon}{\epsilon-1}[\ ]^{\frac{1}{\epsilon}} + \frac{\epsilon-1}{\epsilon} C_t(j)^{-\frac{1}{\epsilon}}$

$$\max_{c_t(j)} \left[\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$$ 最大化消费

$$s.t.\ \ X_t = \int_0^1 P_t(j) C_t(j) dj$$ 预算约束

FOC:

总支出 = 消费　　预算约束.

$$\{C_t(j)\}: \quad C_t(j)^{-\frac{1}{\epsilon}} C_t^{\frac{1}{\epsilon}} = \xi_t P_t(j)$$

求出 $C_t(j)$ 需求方程

$C_t = [\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}} dj]^{\frac{\epsilon}{\epsilon-1}}$

Solution

$\ln C_t(i) - \ln C_t(j) = -\varepsilon \ln P_t(i) + \varepsilon \ln P_t(j)$

$$\frac{C_t(i)}{C_t(j)} = (\frac{P_t(i)}{P_t(j)})^{-\epsilon}$$

$\ln \frac{C_t(i)}{C_t(j)} = -\varepsilon \ln \frac{P_t(i)}{P_t(j)}$

$$C_t(j) = C_t(i)\left(\frac{P_t(i)}{P_t(j)}\right)^{\epsilon}$$

$-\varepsilon \downarrow$

plug into

$$X_t = \int_0^1 P_t(j)C_t(j)dj$$

$= \int_0^1 P_t(j)\, C_t(i) \left(\frac{P_t(i)}{P_t(j)}\right)^{\varepsilon} dj$

$$= C_t(i)P_t(i)^{\epsilon} \int_0^1 P_t(j)^{1-\epsilon}dj$$

get

$$C_t(i) = P_t(i)^{-\epsilon} \frac{X_t}{\int_0^1 P_t(j)^{1-\epsilon}dj}$$

Solution

$$C_t(i) = P_t(i)^{-\epsilon} \frac{X_t}{\int_0^1 P_t(j)^{1-\epsilon} dj}$$

plug into definition of C_t

$$\begin{aligned}
C_t &= \left[\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \\
&= \left[\int_0^1 \left(P_t(i)^{-\epsilon} \frac{X_t}{\int_0^1 P_t(j)^{1-\epsilon} dj} \right)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \\
&= \frac{X_t}{\int_0^1 P_t(j)^{1-\epsilon} dj} \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{\epsilon}{\epsilon-1}} \\
&= \frac{X_t}{\left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}} \quad C_t\text{的价格}
\end{aligned}$$

of which the last equation comes from that i and j are inter-changeable in the last step.

Solution

From

$$C_t = \frac{X_t}{\left(\int_0^1 P_t(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}}$$

combining with definition

$$X_t = P_t C_t$$

we know that

$$\boxed{P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}}$$

Back again into our previous equation

$$C_t(i) = P_t(i)^{-\epsilon} \frac{X_t}{\int_0^1 P_t(j)^{1-\epsilon} dj} = P_t(i)^{-\epsilon} \frac{X_t}{P_t^{1-\epsilon}}$$
$$= \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} \frac{X_t}{P_t} = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t$$

Producers

- Demand for variety i

$$C_t(i) = P_t(i)^{-\epsilon} \frac{X_t}{\int_0^1 P_t(j)^{1-\epsilon} dj} = P_t(i)^{-\epsilon} \frac{X_t}{P_t^{1-\epsilon}}$$
$$= \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} \frac{X_t}{P_t} = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t$$

Constant Return to Scale

- Produce variety CRS with labor

Supply $$Y_t(i) = A_t N_t(i)$$

- Output must equal consumption for each variety:

Demand $$Y_t(i) = C_t(i)$$

Producers: Factor demand

- Firms demands labor to minimize cost subject to production constraint

$\mathcal{L}: w_t N_t(i) + \phi_t (Y_t(i) - A_t N_t(i))$

$$\min_{N_t(i)} w_t N_t(i) + \phi_t(Y_t(i) - A_t N_t(i))$$

$\frac{\partial \mathcal{L}}{\partial N_t(i)} = w_t - \phi_t A_t = 0 \Rightarrow \phi_t = \frac{w_t}{A_t}.$

- ϕ_t is real marginal cost, w is the real wage

$$\{N_t(i)\}: \quad \phi_t = \overset{MC_t}{mc_t} = w_t / A_t$$

real marginal cost is the same for all firms.

Producers: Calvo Assumption

firm 由静态到动态

- Calvo (1983) pricing assumption: Each firm resets price each period with iid probability $1-\theta$.
  - 大数定律 By LLN, fraction that reset is $1-\theta$ and fraction costant is θ.
  - Average price duration follows geometric distribution with mean duration $\frac{1}{1-\theta}$.
- Is the world Calvo?
  - Literally, no.
  - But it could be a decent approximation
- Literature on "menu cost" models where there is an inaction region due to fixed cost of changing price.
  - Initial literature: Much more flexible than Calvo.
  - Recent literature: To match micro-pricing facts, need large and infrequent firm-level MC shocks, which looks close to Calvo.

Producers: the pricing decision

- Firms that adjust prices choose $P_t(i)$,$Y_t(i)$,$N_t(i)$ to maximize expected discounted profits and demand.
- Firms takes into account the possibility they will have to keep this price in the future. (Different with simple RBC)
- For those who re-optimize [there are other terms in firms' objective function omitted here for they are not related to $P_t(i)$]:

消费

$$\max_{P_t(i)} E_t \sum_{s=0}^{\infty} \theta^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[\frac{P_t(i)}{P_{t+s}} Y_{t+s}(i) - \phi_{t+s} Y_{t+s}(i) \right]$$

$$s.t.\ \ C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t \qquad Y_{t+s}(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}$$

$$and\ \ C_t(i) = Y_t(i)\ \ and\ \ C_t = Y_t$$

$$\mathcal{L} = \mathbb{E}_t \sum_{s=0}^{\infty} \theta^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left(\frac{P_t(i)}{P_{t+s}} - \phi_{t+s} \right) \left(\frac{P_t(i)}{P_{t+s}} \right)^{-\epsilon} Y_{t+s}$$

Optimal Price re-setting

$$\frac{\partial \mathcal{L}}{\partial P_t(i)} = \mathbb{E}_t \sum_{s=0}^{\infty} \theta^s \Lambda_{t+s} Y_{t+s} \left((1-\epsilon) \left(\frac{1}{P_{t+s}}\right)^{1-\epsilon} P_t(i)^{-\epsilon} + \phi_{t+s}\, \epsilon \frac{P_t(i)^{-\epsilon-1}}{(P_{t+s})^{-\epsilon}} \right)$$

$= 0$ $\quad 1-\epsilon$

Solve it, we get: $\Rightarrow \quad \frac{\epsilon}{\epsilon-1} P_{t+s} \phi_{t+s}$

- If $\theta = 0$, no stickiness and this collapse to flex price model.

$$P_t^* = (1+\mu)\phi_t P_t, \quad \mu = \frac{1}{\epsilon - 1}$$

- If $\theta > 0$, then the optimal reset price is a markup over a weighted average of expected future marginal costs:

名义边际成本

$$P_t^* = (1+\mu) \frac{E_t \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} Y_{t+s} P_{t+s}^{\epsilon-1} P_{t+s} \phi_{t+s}}{E_t \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} Y_{t+s} P_{t+s}^{\epsilon-1}} \quad \text{(权重)}$$

re-define $\Lambda_{t,t+s} = \beta^s \frac{\lambda_{t+s}}{\lambda_t}$ is the stochastic discount factor

Price Dynamics with Calvo

- Assume symmetric model, so fraction $1-\theta$ of firms adjust to P_t^* and fraction θ keep $P_{t-1}(i)$

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$$

不重置 重置

$$\begin{aligned}
&= \left[\theta \int_0^1 P_{t-1}(i)^{1-\epsilon} di + (1-\theta) \int_0^1 (P_t^*)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}} \\
&= \left[\theta \left[\int_0^1 P_{t-1}(i)^{1-\epsilon} di\right]^{\frac{1-\epsilon}{1-\epsilon}} + (1-\theta) \int_0^1 (P_t^*)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}} \\
&= \left[\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}
\end{aligned}$$

- Price index P_t is geometric average of P_{t-1} and P_t*.
- Recursive formulation is part of why Calvo is so tractable.

Inflation Dynamics with Calvo

- Divide by P_{t-1} to get inflation between $t-1$ and t, Π_t

通胀

$$\Pi_t = \frac{P_t}{P_{t-1}} = \left[\theta + (1-\theta)(\frac{P_t^*}{P_{t-1}})^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

- Calvo pricing implies a partial adjustment mechanism:
  - If $P_t^* = P_{t-1}$, $\Pi_t = 1$
  - If $P_t^* > P_{t-1}$, $\Pi_t > 1$ and $P_t \neq P_{t-1}$.

Completing the Model

- Aggregate output is

$$N_t = \int_0^1 N_t(i) di$$
$$N_t = \int_0^1 \frac{Y_t(i)}{A_t} di = \frac{Y_t}{A_t} \int_0^1 (\frac{P_t(i)}{P_t})^{-\epsilon} di$$
$$Y_t = A_t N_t \frac{1}{\int_0^1 (\frac{P_t(i)}{P_t})^{-\epsilon} di}$$

  - Term in fraction is loss in output due to misallocation caused by price dispersion.
  - Creates welfare costs of inflation, but it is second order and drops out of log-linearization.

A slightly detour: The labor wedge

- Because of CRS, nominal marginal cost is:

$$MC_t = \frac{W_t}{Y_t(i)/N_t(i)} = \frac{W_t}{A_t}$$

实际边际成本 $$mc_t = \frac{MC_t}{P_t}$$

- The labor wedge
  - Note that for firm with markup $\mu_t(i)$,

$$1 + \mu_t(i) = \frac{P_t(i)/P_t}{mc_t}$$

  - In flexible price equilibirum, $P_t(i)/P_t = 1$ so

$$1 + \mu_t(i) = 1/mc_t = \frac{Y_t/N_t}{W_t/P_t} = \frac{MPL_t}{MRS_t}$$

Monetary Policy Rule

We will later introduce central bank policy rule directly in log-linearized form

New Keynesian Model Equilibrium

Definition

A symmetric equilibrium is an allocation $\{C_{t+s}, N_{t+s}, Y_{t+s}\}_{s=0}^{\infty}$ and set of prices $\{P_{t+s}^{*}, P_{t+s}, W_{t+s}, Q_{t+s}\}_{s=0}^{\infty}$ along with exogenous processes $\{A_{t+s}, Z_{t+s}, v_{t+s}\}_{s=0}^{\infty}$ such that:

1. Households optimize: Euler, labor-leisure, (money demand in background as the central bank chooses Q_{t+s}, putting B_{t+s} in background as well.)

2. Firm optimize: Price index follows dynamic Calvo formulation; Intermediate reset prices are chosen optimally given nominal marginal cost: $MC_t = \frac{W_t}{Y_t(i)/N_t(i)} = \frac{W_t}{A_t}$.

3. Central bank follows interest rate rule with shock v_t.

4. Labor and goods (and bond) markets clear.

Log Linearization Strategy

(sticky price)
NK output − flexiable price output.
$\hat{y}_t - \hat{y}_t^n$

- Phillips curve should be function of output gap, so want to write whole model as functions of output gap.
- Strategy:
  1. Log-linearize Model around zero-inflation steady state. (IS-PC-MP)
  2. Log-linearize Flexible Price Equilibrium.
  3. Difference to get equilibrium in terms of output gap.

Log Linearization: supply block

$$\frac{W_t}{P_t} = \frac{N_t^{\varphi}}{C_t^{-\sigma}}$$

$$Y_t = A_t N_t \frac{1}{\int_0^1 (\frac{P_t(i)}{P_t})^{-\epsilon} di}$$

$$\Pi_t = \frac{P_t}{P_{t-1}} = \left[\theta + (1-\theta)(\frac{P_t^*}{P_{t-1}})^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

$$P_t^* = (1+\mu)\frac{E_t \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} Y_{t+s} P_{t+s}^{\epsilon-1} P_{t+s} \phi_{t+s}}{E_t \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} Y_{t+s} P_{t+s}^{\epsilon-1}}$$

Labor-leisure:

$$\hat{w}_t - \hat{p}_t = \varphi \hat{n}_t + \sigma \hat{c}_t$$

Production function: [one term missing here,see appendix slides]

$$\hat{y}_t = \hat{a}_t + \hat{n}_t$$

Log Linearization: Inflation and Reset Prices

Details in appendix slides

- Key trick: Zero inflation steady state
- The price index can be log-linearized to get

$$\hat{p_t} = \theta \hat{p_{t-1}} + (1-\theta)\hat{p_t^*}$$
$$\hat{\pi}_t = (1-\theta)(\hat{p_t^*} - \hat{p_{t-1}})$$

$P_t = [\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{*1-\varepsilon}]^{\frac{1}{1-\varepsilon}}$

Log Linearization: Phillips Curve

- We will get an expectation-augmented Phillips curve [see appendix slides for details]:

  边际成本变动更少的部分进入 $\hat{\pi}_t$.

$$\hat{\pi}_t = \lambda \hat{\phi}_t + \beta E_t \left\{ \hat{\pi_{t+1}} \right\}$$

where

$$\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta} \quad \theta \uparrow, \lambda \downarrow$$

- Inflation is equal to expected future inflation plus the deviation of marginal cost from its steady state level.
- Iterating forward

$$\hat{\pi}_t = \lambda E_t \left\{ \sum_{s=0}^{\infty} \beta^s \hat{\phi}_{t+s} \right\} = \lambda E_t \left\{ \sum_{s=0}^{\infty} \beta^s \hat{mc}_{t+s} \right\}$$

  - Inflation is the present discounted value of future marginal cost/markup deviations from steady state.

Log Linearization: Real Marginal Costs

Real marginal cost:

mc_t的百分比变动而非MC_t的百分比变动

$$\hat{mc}_t = \hat{w}_t - \hat{p}_t - \hat{a}_t$$

Combine labor-leisure, production function, and $\hat{c}_t = \hat{y}_t$:

$\hat{w}_t - \hat{p}_t = \varphi \hat{n}_t + \sigma \hat{c}_t$

$= \varphi(\hat{y}_t - \hat{a}_t) + \sigma \hat{c}_t$

$$\hat{w}_t - \hat{p}_t = (\sigma + \varphi)\hat{y}_t - \varphi \hat{a}_t$$

$= (\sigma + \varphi)\hat{y}_t - \varphi \hat{a}_t$

Consequently

$$\hat{mc}_t = (\sigma + \varphi)\hat{y}_t - (1 + \varphi)\hat{a}_t$$

Log Linearization: Flexible Price Equilibrium

$$Y_t^n = A_t N_t^n$$
$$\frac{W_t^n}{P_t^n} = \frac{A_t}{1+\mu}$$
$$\frac{W_t^n}{P_t^n} = \frac{(N_t^n)^\varphi}{(C_t^n)^{-\sigma}}$$
$$Y_t^n = C_t^n$$

$$\frac{A_t}{1+\mu} = \frac{(N_t^n)^\varphi}{(C_t^n)^{-\sigma}}$$

$$A_t = (1+\mu)\left(\frac{Y_t^n}{A_t}\right)^\varphi \cdot Y_t^{n\,\sigma}$$

Combine to get:

$$A_t^{1+\varphi} = (1+\mu)(Y_t^n)^{\sigma+\varphi}$$

$$A_t^{1+\varphi} = (1+\mu)(Y_t^n)^{\sigma+\varphi}$$
$$(\sigma+\varphi)\hat{y}_t{}^n = (1+\varphi)\hat{a}_t$$

Real Marginal Costs in Terms of Output Gap

$$\tilde{y}_t = \hat{y}_t - \hat{y}_t^n = \frac{y_t - y}{y} - \frac{y_t^n - y}{y} = \frac{y_t - y_t^n}{y}.$$

- Combine:

$$\hat{mc}_t = (\sigma + \varphi)\hat{y}_t - (1 + \varphi)\hat{a}_t$$
$$(\sigma + \varphi)\hat{y}_t{}^n = (1 + \varphi)\hat{a}_t$$

$\hat{y}_t - \hat{y}_t^n$

NK条件下产出 − flexiable price产出

to write real marginal costs in terms of output gap $\tilde{y}_t$:

$$\hat{mc}_t = (\sigma + \varphi)(\hat{y}_t - \hat{y}_t{}^n) = (\sigma + \varphi)\tilde{y}_t$$

- Real marginal costs go up (and markups go down) when the output gap is high.

del. (*)

  - To produce more than under flex prices, markup must be lower.
  - Marginal costs high because need to hire more workers, bidding up real wage.
  - Stronger when Intertemporal Elasticity of Substitution and labor supply elasticity are low.

The New Keynesian Phillips Curve

- Plug back into the Phillips curve $\hat{\pi}_t = \lambda \hat{mc}_t + \beta E_t \{\hat{\pi_{t+1}}\}$:

$$\hat{\pi}_t = \kappa \tilde{y}_t + \beta E_t \{\hat{\pi_{t+1}}\}$$

Phillips curve 斜率 (slope)

where $\kappa = \lambda(\sigma + \varphi)$.

  - This is the New Keynesian Phillips Curve: an expectations augmented Phillips curve written in terms of the output gap.
- Solving forward,

$$\hat{\pi}_t = \kappa E_t \sum_{s=0}^{\infty} \beta^s \tilde{y_{t+s}}$$

  - Inflation is an increasing function of future output gaps.

Log Linearization: Demand

$$\frac{1}{R_t} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}}$$

$$-\hat{r}_t = +(\hat{z}_t - E_t \hat{z}_{t+1})$$
$$-\sigma(\hat{c}_t - E_t \hat{c}_{t+1})$$
$$-(\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}) = \cdots$$

$$Q_t = 1/R_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}}$$

$$Y_t = C_t$$

$$\sigma(\hat{c}_t - E_t \hat{c}_{t+1}) + (\hat{i}_t - E_t\{\hat{\pi}_{t+1}\}) - \hat{z}_t - E_t \hat{z}_{t+1}).$$

$$\hat{c}_t = -\frac{1}{\sigma}(\cdots\cdots)$$

- Log-linearize Euler around zero-inflation.

$$\hat{c}_t = -\frac{1}{\sigma}(\hat{i}_t - E_t\{\hat{\pi_{t+1}}\} - (\hat{z}_t - E_t \hat{z_{t+1}})) + E_t\{\hat{c_{t+1}}\}$$

- Combine with market clearing and use :

$$\hat{y}_t = -\frac{1}{\sigma}(\hat{i}_t - E_t\{\hat{\pi_{t+1}}\} - (\hat{z}_t - E_t \hat{z_{t+1}})) + E_t\{\hat{y_{t+1}}\}$$

- This is the dynamic IS curve. It relates output to future expectations of output and the real interest rate.

The Natural Rate of Interest

- Define the natural rate of interest $\hat{r_t}^n$ as the real interest rate that would prevail when output is equal to its flexible level (natural level of output):

$$\hat{y_t}^n = -\frac{1}{\sigma}(\hat{r_t}^n - (\hat{z}_t - E_t \hat{z_{t+1}})) + E_t\{\hat{y_{t+1}}^n\}$$

- Recall $\hat{y_t}^n = (\frac{1+\varphi}{\sigma+\varphi})\hat{a}_t$ so:

$$\hat{r_t}^n = \sigma(\frac{1+\varphi}{\sigma+\varphi})E_t\{\hat{a_{t+1}} - \hat{a}_t\} + (\underbrace{\hat{z}_t}_{\text{demand shock}} - E_t \hat{z_{t+1}})$$

  - If a_t follows an AR(1) and grows today, it will be expected to decline between today and tomorrow due to mean reversion.
  - So positive tech shock causes real interest rate to fall by standard RBC logic.

IS Curve in Terms of the Output Gap

$\hat{r}_t = \hat{i}_t - \mathbb{E}_t\{\hat{\pi}_{t+1}\}.$

$$\hat{y}_t = -\frac{1}{\sigma}(\underbrace{\hat{i}_t - E_t\{\widehat{\pi_{t+1}}\}}_{\|} - (\hat{z}_t - E_t\widehat{z_{t+1}})) + E_t\{\widehat{y_{t+1}}\} \qquad (1)$$

转换为 natural

$$\hat{y}_t{}^n = -\frac{1}{\sigma}(\hat{r}_t{}^n - (\hat{z}_t - E_t\widehat{z_{t+1}})) + E_t\{\widehat{y_{t+1}}{}^n\} \qquad (2)$$

(1)-(2):

$$\tilde{y}_t = -\frac{1}{\sigma}E_t\left\{\overbrace{\hat{i}_t - \widehat{\pi_{t+1}} - \widehat{r_t^n}}^{=\hat{r}_t - \hat{r}_t^n}\right\} + E_t\{\widetilde{y_{t+1}}\}$$

Iterating forward, the current output gap depends negatively on the gap between the real interest rate and the natural rate of interest:

$$\tilde{y}_t = -\frac{1}{\sigma}E_t\left\{\sum_{s=0}^{\infty}(\widehat{r_{t+s}} - \widehat{r_{t+s}^n})\right\}$$

The Three Equation Model

In sum, the NK model boils down to three equations:

$$\tilde{y}_t = -\frac{1}{\sigma} E_t \left\{ \hat{i}_t - \hat{\pi_{t+1}} - \hat{r_t^n} \right\} + E_t\{\tilde{y_{t+1}}\}$$
$$\hat{\pi}_t = \kappa \tilde{y}_t + \beta E_t \left\{ \hat{\pi_{t+1}} \right\}$$
$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_y \tilde{y}_t + v_t$$

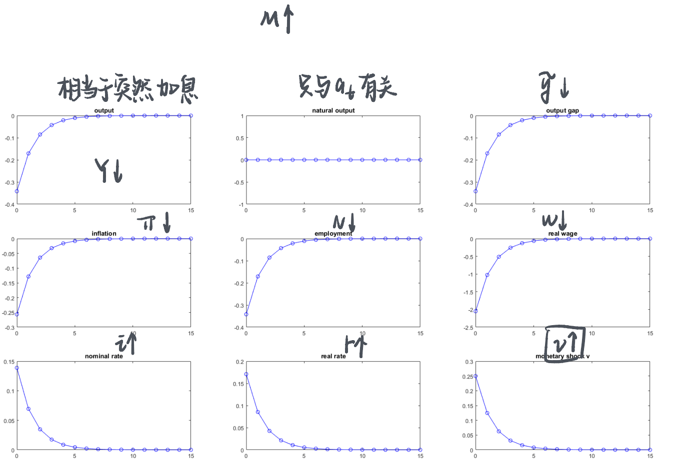
with three unknowns: $\hat{i}_t$, $\tilde{y}_t$, and $\hat{\pi}_t$ and an exogenous driving process for the natural rate:

$$\hat{r_t}^n = \sigma\left(\frac{1+\varphi}{\sigma+\varphi}\right) E_t\{\hat{a_{t+1}} - \hat{a}_t\} + (\hat{z}_t - E_t \hat{z_{t+1}})$$

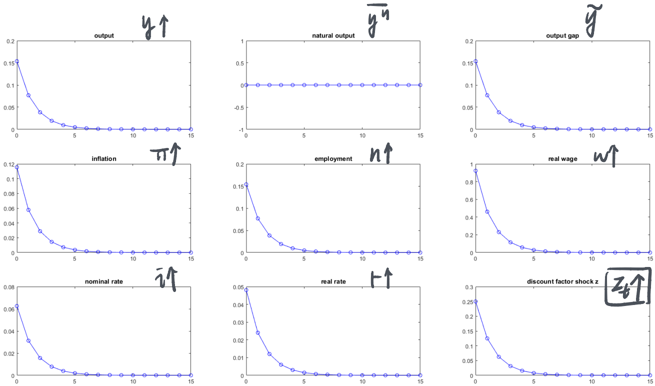
Calibration Parameters

- $\beta = 0.99$
- $\alpha = 0.33$ (Back to more general case). $\sigma = 1$ (Log utility), $\phi = 5$ (Frisch elasticity of labor supply: 0.2)
- $\epsilon = 9$ (Steady state markup: 12.5 percent)
- $\theta = 0.75$ (Average price duration: four quarters)
- $\phi_\pi = 1.5, \phi_y = 0.125.$
- $\rho_v = 0.5$,$\rho_z = 0.5$,$\rho_a = 0.9$

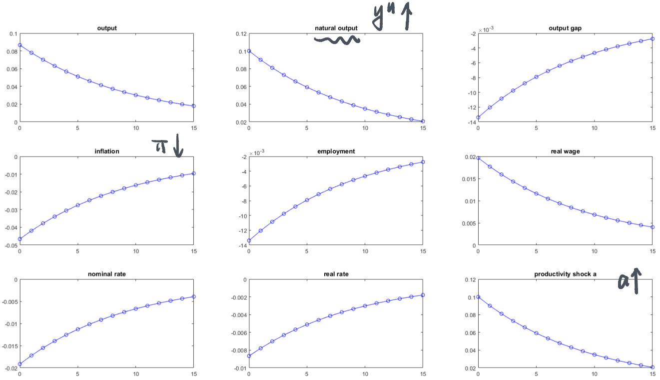
Impulse Response: Monetary Shock



Impulse Response: Discount Rate Shock



Impulse Response: Tech Shock



Limitation of "Simple" NK Models

- "Simple" New Keynesian models like the one we have studied have limitations.
  - For starters, no capital!
  - No inflation persistence.
  - No wage stickiness (which makes marginal costs sticky and prices stickier).
  - Little amplification of Calvo friction (price level has adjusted once the time most firms have reset price).
- Big literature adding many features back into basic NK model.

Medium-Scale NK Models

- "State of the Art" for NK literature (absent financial frictions) is "Medium-Scale" models of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007).
- Goal: Quantitative parameterized model that can be used for ...
  - Forecasting
  - But also, structural model so can be use for shock decompositions, policy experiments, and other counterfactuals.
- Consequently, as add features to RBC to fit data, add features to NK to fully fit data.
  - Then use GMM or Bayesian methods to fit model to match VAR impulse responses in simulated data.
- Parameterized models do a remarkably good job
  - As good at forecasting over 1-2 years as VAR
  - Allow for shock decompositions and counterfactuals
  - Used at many central banks

Medium-Scale NK Models: Smets and Wouters

- Example: Smets and Wouters (2007) has
  - Calvo pricing and inflation indexation
  - Calvo wages
  - Capital and investment adjustment costs
  - Habit formation in consumption
  - Variable capital utilization
  - Fixed costs in production
  - Strategic complementarity in price setting
- And seven shocks:
  - TFP
  - Risk premium shock
  - Investment technology shocks
  - Wage markup shocks
  - Price markup shocks
  - Government spending shock
  - Monetary policy shock
- When people say "New Keynesian" they mean these models

Does the NKPC Hold in the Data?

- NK Phillips Curve is at the center of the NK model
- Final critique: Does it hold in the data?

$$\hat{\pi}_t = \kappa \tilde{y}_t + \beta E_t\{\hat{\pi_{t+1}}\}$$

  - How do we measure inflation expectations?
  - How do we measure "natural rate of output" and output gap?
- One approach: Are markups countercyclical?

APPENDIX

Connecting inflation with the optimal reset price

We need to log-linearize the following equation connecting the current price index with the lagged price index and the optimal reset price.

$$\frac{P_t}{P_{t-1}} = \left[\theta + (1-\theta)(\frac{P_t^*}{P_{t-1}})^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

$$(\frac{P_t}{P_{t-1}})^{1-\epsilon} = \theta + (1-\theta)(\frac{P_t^*}{P_{t-1}})^{1-\epsilon}$$

Take the total derivative around the steady state:

$$(1-\epsilon)(\frac{P_t}{P_{t-1}})^{-\epsilon}(dp_t - dp_{t-1}) = (1-\theta)(1-\epsilon)(\frac{P_t^*}{P_{t-1}})^{-\epsilon}(dp_t^* - dp_{t-1})$$

since we are at the steady state, the term in blue equals one.

$$p\hat{p}_t - p\hat{p_{t-1}} = (1-\theta)(p\hat{p_t^*} - p\hat{p_{t-1}})$$

Connecting inflation with the optimal reset price

$$\hat{pp_t} - \hat{pp_{t-1}} = (1-\theta)(\hat{pp^*_t} - \hat{pp_{t-1}})$$

in which we use the definition of $\hat{x_t} = \frac{x_t - x}{x} = \frac{dx_t}{x}$. x is the steady state value for a certain variable.
Then we get

$$\hat{\pi}_t = (1-\theta)(\hat{p^*_t} - \hat{p_{t-1}})$$

Appendix: price dispersion in production

We need to log-linearize the following term

$$Y_t = A_t N_t \frac{1}{\int_0^1 (\frac{P_t(i)}{P_t})^{-\epsilon} di}$$

we write in the following form:

$$\hat{y_t} = \hat{a_t} + \hat{n_t} - \hat{d_t}$$

of which d_t corresponds to the price dispersion term.

Appendix: Log Linearization

- Taylor Expansion of $f(x)$ around x^*

$$f(x) = f(x^*) + \frac{f'(x^*)}{1!}(x - x^*) + \frac{f''(x^*)}{2!}(x - x^*)^2 + \cdots$$

- For a sufficiently smooth function, the higher order derivatives will be small.

$$f(x) = f(x^*) + f'(x^*)(x - x^*)$$

- Log Linearization: log it first, Taylor expand it then.

Appendix: Log Linearization

- Example

$$y_t = A_t k_t^{\alpha} N_t^{1-\alpha}$$

- Take logs:

$$lny_t = lnA_t + \alpha lnk_t + (1-\alpha)lnN_t$$

- Do the Taylor expansion around the steady state values:

$$\begin{aligned} lny^* + \frac{1}{y^*}(y_t - y^*) = lnA^* + \frac{1}{A^*}(A_t - A^*) + \\ \alpha lnk^* + \alpha\frac{1}{k^*}(k_t - k^*) + \\ (1-\alpha)lnN^* + (1-\alpha)\frac{1}{N^*}(N_t - N^*) \end{aligned}$$

- With hat indicating percentage deviation from the steady state values, we get

$$\hat{y_t} = \hat{A}_t + \alpha\hat{k_t} + (1-\alpha)\hat{N_t}$$

Appendix: price dispersion in production

We expand the exponential function around the steady state where $P_t(i) = P_t$.

$$(\frac{P_t(i)}{P_t})^{1-\epsilon} = exp\big[(1-\epsilon)(\hat{p_t(i)} - \hat{p}_t)\big]$$

$$= 1 + (1-\epsilon)(\hat{p_t(i)} - \hat{p}_t) + \frac{(1-\epsilon)^2}{2}(\hat{p_t(i)} - \hat{p}_t)^2$$

Since we have $\int_0^1 (\frac{P_t(i)}{P_t})^{1-\epsilon} di = 1$. A second order linear approximation will lead to:

$$E_i\{\hat{p_t(i)} - \hat{p}_t\} = \frac{\epsilon - 1}{2} E_t\{(\hat{p_t(i)} - \hat{p}_t)^2\}$$

$$(\frac{P_t(i)}{P_t})^{-\epsilon} = 1 - \epsilon(\hat{p_t(i)} - \hat{p}_t) + \frac{\epsilon^2}{2}(\hat{p_t(i)} - \hat{p}_t)^2$$

Combine the above two equations:

$$\int_0^1 (\frac{P_t(i)}{P_t})^{-\epsilon} di = 1 + \frac{\epsilon}{2} E_t\{(\hat{p_t(i)} - \hat{p}_t)^2\}$$

Appendix: price dispersion in production

$$\int_0^1 (\frac{P_t(i)}{P_t})^{-\epsilon} di = 1 + \frac{\epsilon}{2} E_t\{(\hat{p_t(i)} - \hat{p}_t)^2\}$$
$$\approx 1 + \frac{\epsilon}{2} var_i\{\hat{p_t(i)}\}$$

The second line comes from:

$$\int_0^1 (\hat{p_t(i)} - \hat{p}_t)^2 di \approx \int_0^1 (\hat{p_t(i)} - E_i\{\hat{p}_t\})^2 di \equiv var_i\{\hat{p_t(i)}\}$$

so

$$d_t = log[\int_0^1 (\frac{P_t(i)}{P_t})^{-\epsilon} di] \approx \frac{\epsilon}{2} var_i\{\hat{p_t(i)}\}$$

Appendix: Linearization of the optimal price setting

We know

$$P_t^* = (1+\mu)\frac{E_t \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} Y_{t+s} P_{t+s}^{\epsilon-1} P_{t+s} \phi_{t+s}}{E_t \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} Y_{t+s} P_{t+s}^{\epsilon-1}}$$

Rewrite the denominator and numerator:

$$X_{1,t} = Z_t C_t^{-\sigma} mc_t P_t^{\epsilon} Y_t + \beta\theta E_t X_{1,t+1}$$
$$X_{2,t} = Z_t C_t^{-\sigma} P_t^{\epsilon-1} Y_t + \beta\theta E_t X_{2,t+1}$$

Define

$$x_{1,t} = \frac{X_{1,t}}{P_t^{\epsilon}}$$
$$x_{2,t} = \frac{X_{2,t}}{P_t^{\epsilon-1}}$$

Appendix: Linearization of the optimal price setting

$$x_{1,t} = Z_t C_t^{-\sigma} mc_t Y_t + \beta\theta E_t \frac{X_{1,t+1}}{P_{t+1}^{\epsilon}} (\frac{P_{t+1}}{P_t})^{\epsilon}$$

$$x_{2,t} = Z_t C_t^{-\sigma} Y_t + \beta\theta E_t \frac{X_{2,t+1}}{P_{t+1}^{\epsilon-1}} (\frac{P_{t+1}}{P_t})^{\epsilon-1}$$

In short,

$$x_{1,t} = Z_t C_t^{-\sigma} mc_t Y_t + \beta\theta E_t x_{1,t+1} (1+\pi_{t+1})^{\epsilon} \qquad (3)$$

$$x_{2,t} = Z_t C_t^{-\sigma} Y_t + \beta\theta E_t x_{2,t+1} (1+\pi_{t+1})^{\epsilon-1} \qquad (4)$$

Appendix: Linearization of the optimal price setting

We use $Y_t = C_t$ and take total derivative of (3):

$$x_1 \hat{x_{1,t}} = (1-\sigma) Z Y^{-\sigma} mc Y \hat{Y}_t + Z Y^{1-\sigma} mc \hat{mc}_t + Y^{1-\sigma} mc Z \hat{Z}_t \tag{5}$$

$$+ \epsilon \beta \theta (1+\pi)^{\epsilon - 1} x_1 (1+\pi) E_t \hat{\pi_{t+1}} + \beta \theta (1+\pi)^{\epsilon} x_1 E_t \hat{x_{1,t+1}} \tag{6}$$

of which the percentage deviation of inflation is different from variables like y.

$$log(1+\pi_{t+1}) - log(1+\pi) = log(1 + \frac{1+\pi_{t+1} - (1+\pi)}{1+\pi})$$
$$\approx \frac{\pi_{t+1} - \pi}{1+\pi} = \hat{\pi_{t+1}}$$

Appendix: Linearization of the optimal price setting

We simplify (5-6) at zero inflation steady state $Z = 1$.

$$x_1 \hat{x_{1,t}} = (1-\sigma)Y^{-\sigma} mcY\hat{Y}_t + Y^{1-\sigma} mc\hat{mc}_t + Y^{1-\sigma} mc\hat{Z}_t$$
$$+ \epsilon\beta\theta x_1 E_t \hat{\pi_{t+1}} + \beta\theta x_1 E_t \hat{x_{1,t+1}}$$

and evaluate equation (3) at the steady state we get

$$x_1 = \frac{Y^{1-\sigma} mc}{1-\beta\theta}$$

plug into the previous equation,

$$\hat{x_{1,t}} = (1-\sigma)(1-\beta\theta)\hat{Y}_t + (1-\beta\theta)\hat{mc}_t + (1-\beta\theta)\hat{Z}_t$$
$$+ \epsilon\beta\theta E_t \hat{\pi_{t+1}} + \beta\theta E_t \hat{x_{1,t+1}}$$

Appendix: Linearization of the optimal price setting

$$
\begin{aligned}
\hat{x_{1,t}} &= (1-\sigma)(1-\beta\theta)\hat{Y_t} + (1-\beta\theta)\hat{mc_t} + (1-\beta\theta)\hat{Z}_t \\
&\quad + \epsilon\beta\theta E_t \hat{\pi_{t+1}} + \beta\theta E_t \hat{x_{1,t+1}}
\end{aligned}
$$

Following similar steps, we have

$$
\begin{aligned}
\hat{x_{2,t}} &= (1-\sigma)(1-\beta\theta)\hat{Y_t} + (1-\beta\theta)\hat{Z}_t \\
&\quad + (\epsilon-1)\beta\theta E_t \hat{\pi_{t+1}} + \beta\theta E_t \hat{x_{2,t+1}}
\end{aligned}
$$

Take the difference of the two equations

$$
\hat{x_{1,t}} - \hat{x_{2,t}} = (1-\beta\theta)\hat{mc_t} + \beta\theta E_t \hat{\pi_{t+1}} + \beta\theta E_t[\hat{x_{1,t+1}} - \hat{x_{2,t+1}}] \tag{7}
$$

Appendix: Linearization of the optimal price setting

We have the following from definition of $x_{1,t}, x_{2,t}$:

$$\frac{X_{1,t}}{X_{2,t}} = P_t \frac{x_{1,t}}{x_{2,t}}$$

Thus, we have

$$P_t^* = (1+\mu) P_t \frac{x_{1,t}}{x_{2,t}}$$

$$P_t^* / P_{t-1} = (1+\mu) P_t / P_{t-1} \frac{x_{1,t}}{x_{2,t}}$$

$$1 + \pi_t^* = (1+\mu)(1+\pi_t) \frac{x_{1,t}}{x_{2,t}}$$

$$\hat{\pi_t^*} = \hat{\pi}_t + \hat{x_{1,t}} - \hat{x_{2,t}}$$

We got the following equation in the main slides

$$\hat{\pi}_t = (1-\theta)(\hat{p_t^*} - \hat{p_{t-1}}) = (1-\theta)\hat{\pi_t^*}$$

Appendix: Linearization of the optimal price setting

We now have

$$\hat{\pi_t^*} = \hat{\pi}_t + \hat{x_{1,t}} - \hat{x_{2,t}}$$
$$\hat{\pi}_t = (1-\theta)\hat{\pi_t^*}$$

we get

$$\frac{\theta}{1-\theta}\hat{\pi}_t = \hat{x_{1,t}} - \hat{x_{2,t}}$$

Plug back into equation (7), we get

$$\hat{\pi}_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{mc_t} + \beta E_t \hat{\pi_{t+1}}$$

FINALLY DONE.

Appendix: Linearization of the optimal price setting

We now have

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