Intermediate Microeconomics

Chapter 3: Preferences

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Preference Relations

Comparing two different consumption bundles, (x_1, x_2) and (y_1, y_2), we may have the following preference relations:

1. Strict preference: $(x_1, x_2) \succ (y_1, y_2)$

2. Indifference: $(x_1, x_2) \sim (y_1, y_2)$

3. Weak preference: $(x_1, x_2) \succsim (y_1, y_2)$

These preference relations are ordinal relations; i.e., they state only the order in which bundles are preferred.

Assumption about Preference Relations

Completeness: For any two bundles , (x_1, x_2) and (y_1, y_2), it is always possible to make the statement that either

$$(x_1, x_2) \succsim (y_1, y_2)$$

or

$$(x_1, x_2) \precsim (y_1, y_2)$$

Assumption about Preference Relations

Reflexivity: We assume that any bundle is at least as good as itself

$$(x_1, x_2) \succsim (x_1, x_2)$$

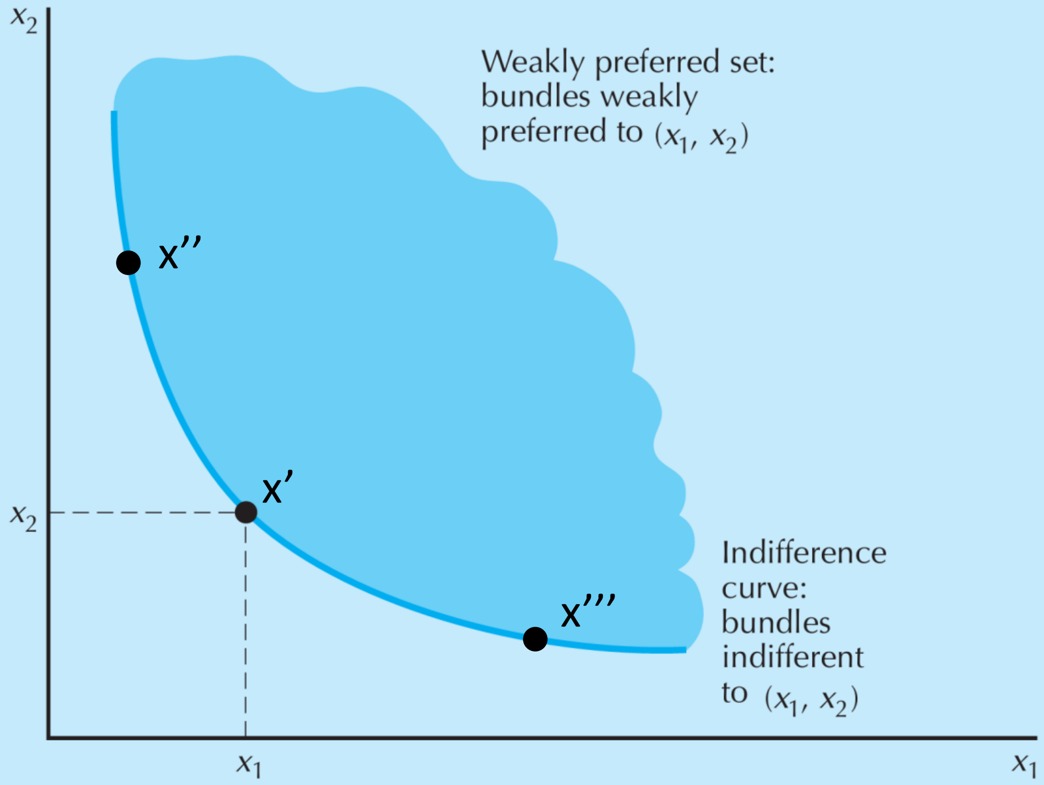
Assumption about Preference Relations

Transitivity: For any two bundles , (x_1, x_2) and (y_1, y_2), it is always possible to make the statement that

If (x_1, x_2) is at least as preferred as (y_1, y_2), and
(y_1, y_2) is at least as preferred as (z_1, z_2), then
(x_1, x_2) is at least as preferred as (z_1, z_2).

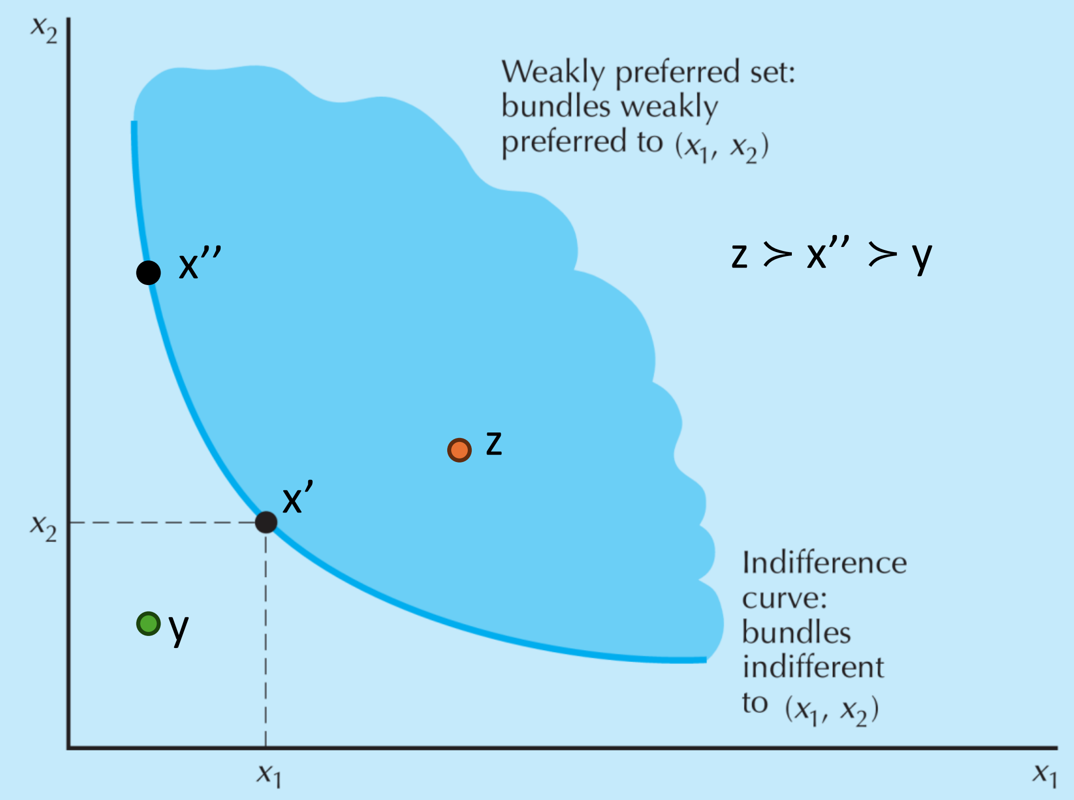
$$(x_1, x_2) \succsim (y_1, y_2) \text{ and } (y_1, y_2) \succsim (z_1, z_2) \quad \rightarrow \quad (x_1, x_2) \succsim (z_1, z_2)$$

Indifference Curves



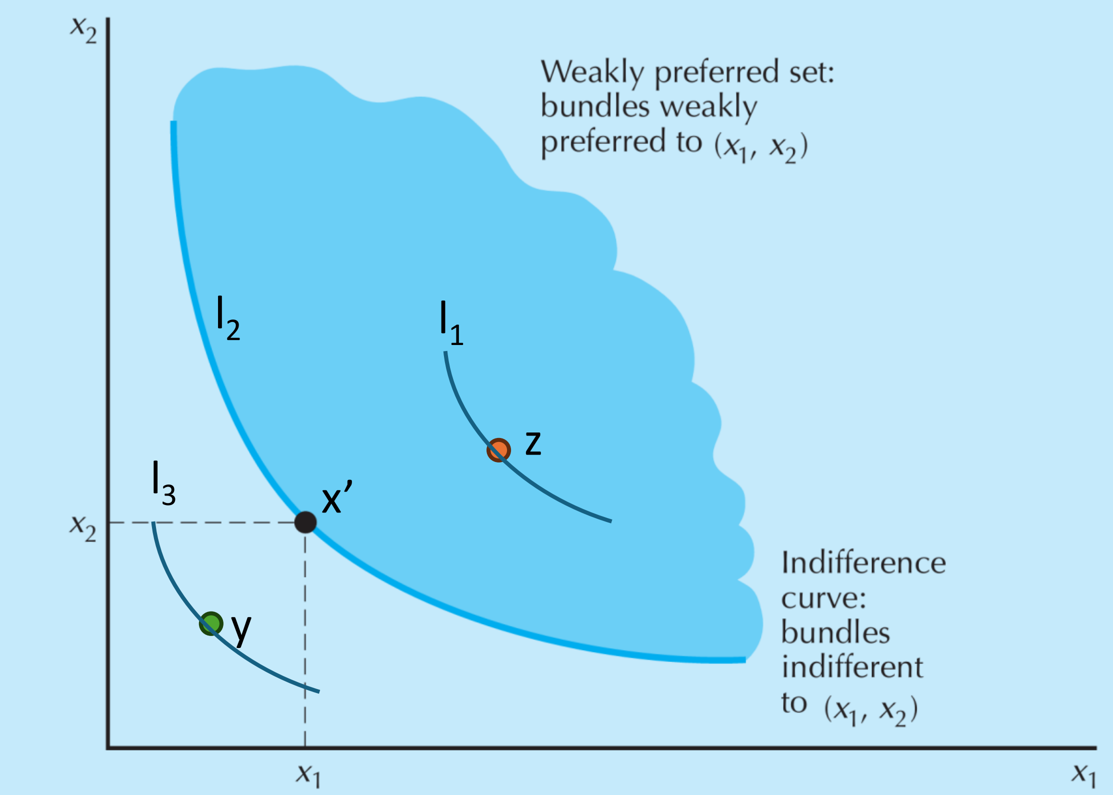
Weakly preferred set. The shaded area consists of all bundles that are at least as good as the bundle (x_1, x_2).

Indifference Curves: More is Preferred to Less



Weakly preferred set. The shaded area consists of all bundles that are at least as good as the bundle (x_1, x_2).

Indifference Curves

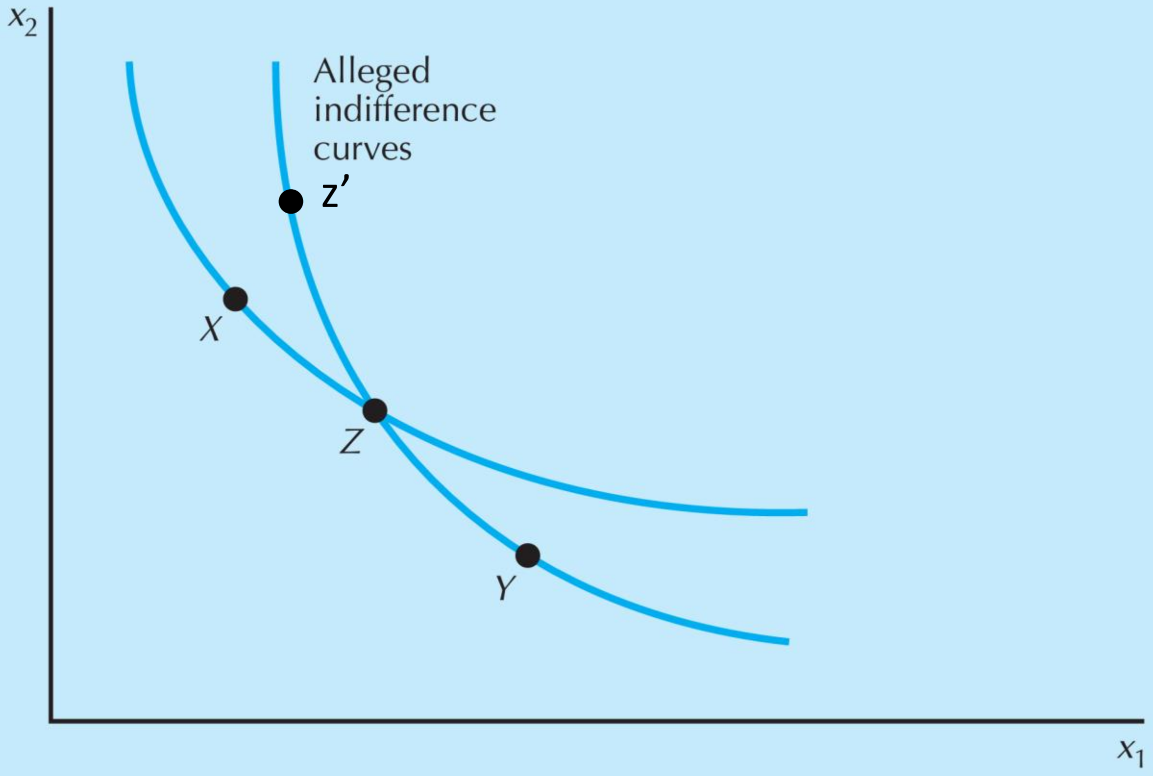


Weakly preferred set. The shaded area consists of all bundles that are at least as good as the bundle (x_1, x_2).

All bundles in I_1 are strictly preferred to all in I_2;

All bundles in I_2 are strictly preferred to all in I_3.

Indifference Curves: No Crossing Property



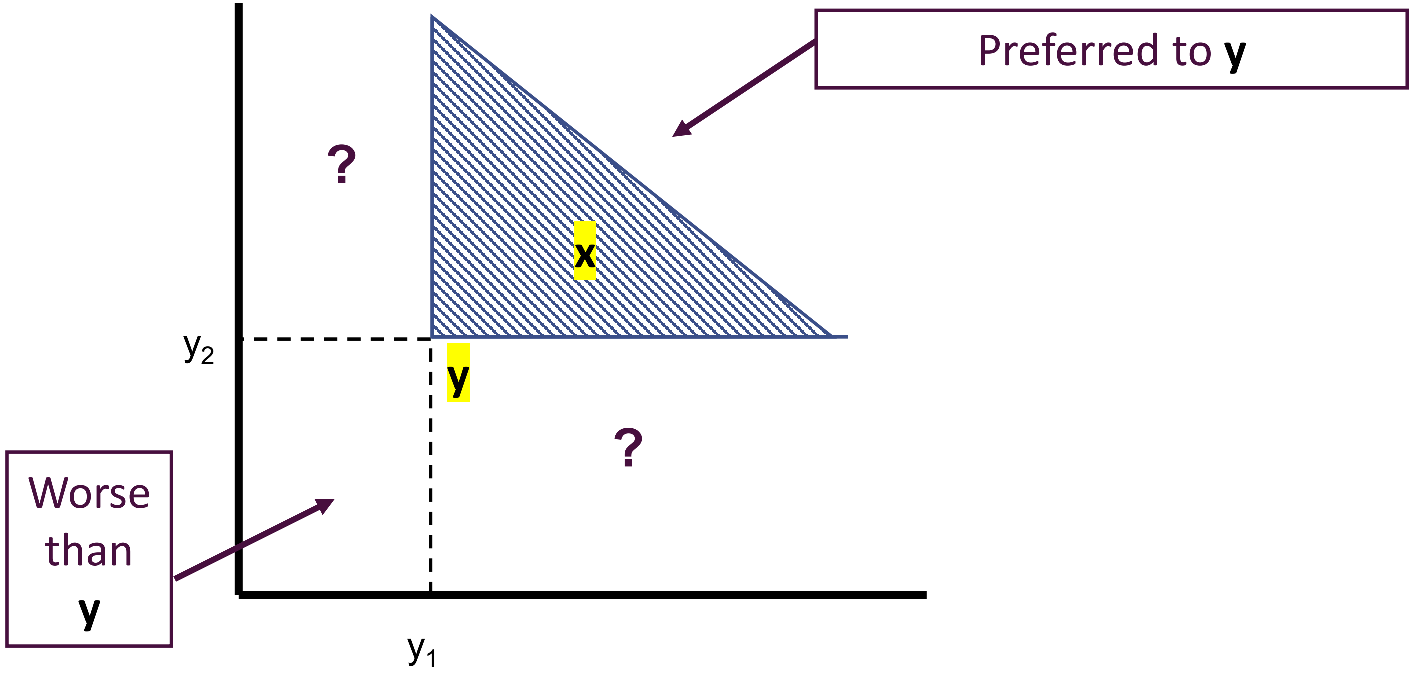
Indifference curves cannot cross. If they did, X, Y, and Z would all have to be indifferent to each other and thus could not lie on distinct indifference curves.

Indifference Curves: Monotone

$(x_1, x_2), (y_1, y_2) \in X$

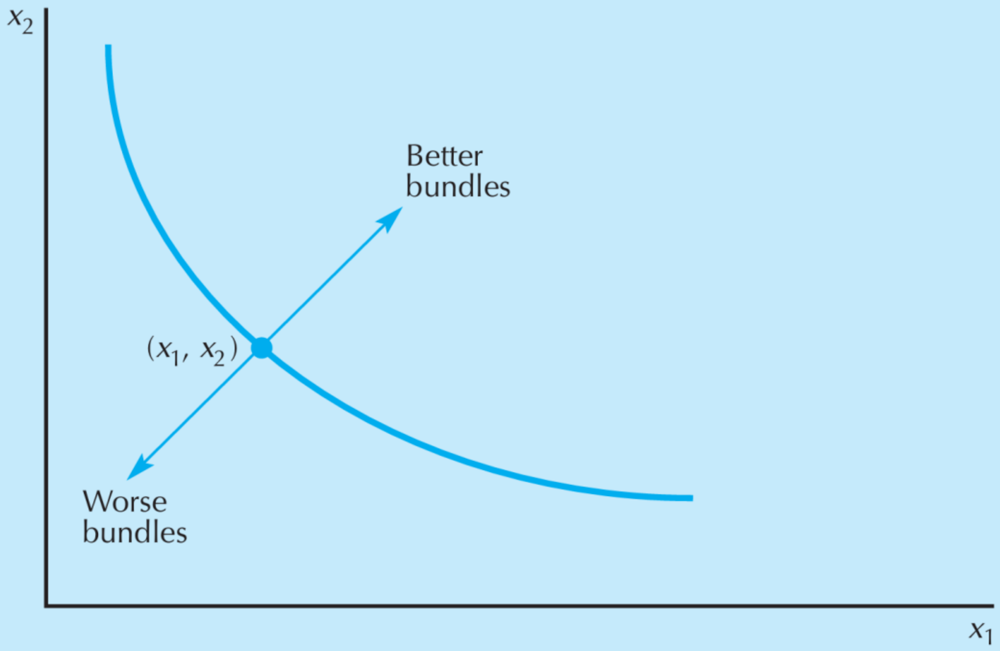
$(x_1, x_2) \geq (y_1, y_2) \quad \Rightarrow \qquad (x_1, x_2) \succ (y_1, y_2)$

Indifference Curves: Monotone



$(x_1, x_2) \geq (y_1, y_2)$ $\qquad$ $(x_1, x_2) \succsim (y_1, y_2)$

Indifference Curves: Monotone



Monotonic preferences. More of both goods is a better bundle for this consumer; less of both goods represents a worse bundle.

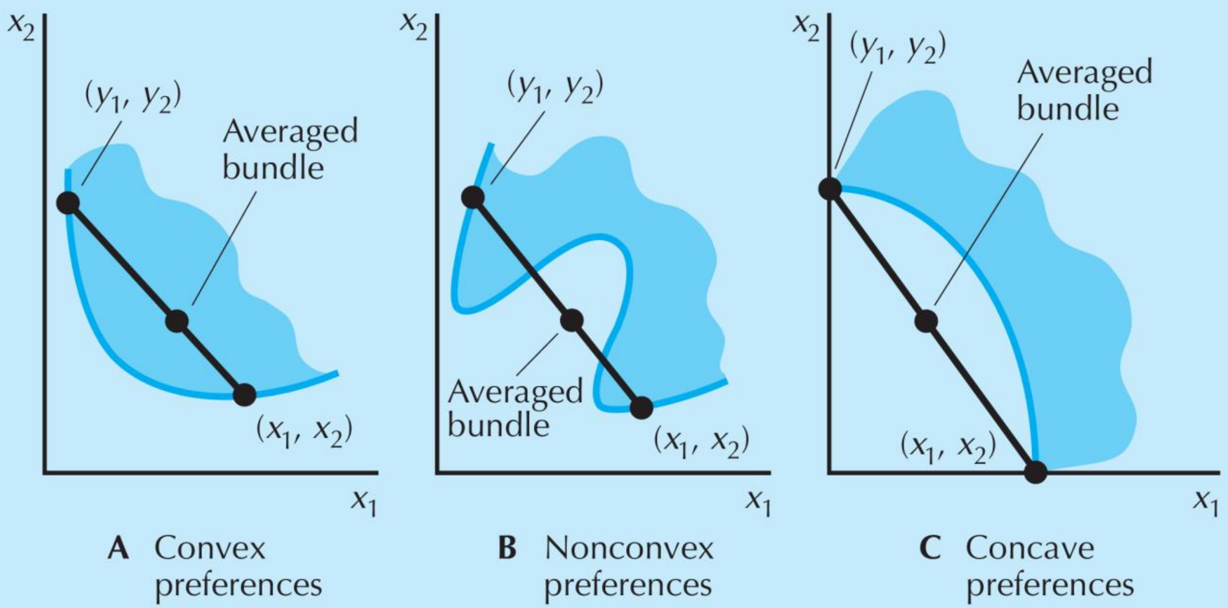
Indifference Curves: Convexity

A set of points is convex if any two points can be joined by a straight line that is contained completely within the set

$$\mathbf{x} \sim \mathbf{y}$$

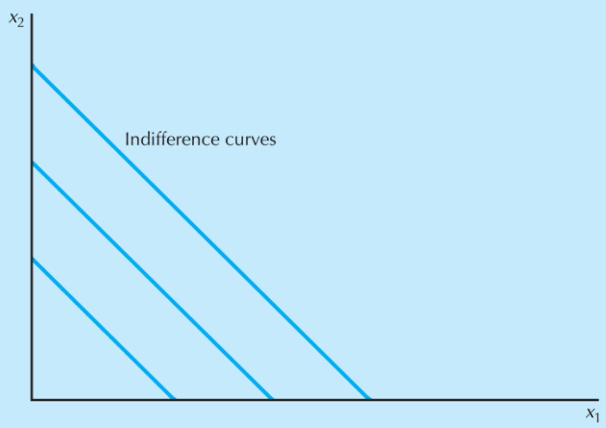
$$\alpha\mathbf{x} + (1-\alpha)\mathbf{y} \succsim \mathbf{x}$$
$$\alpha\mathbf{x} + (1-\alpha)\mathbf{y} \succsim \mathbf{y}$$

Indifference Curves: Convexity

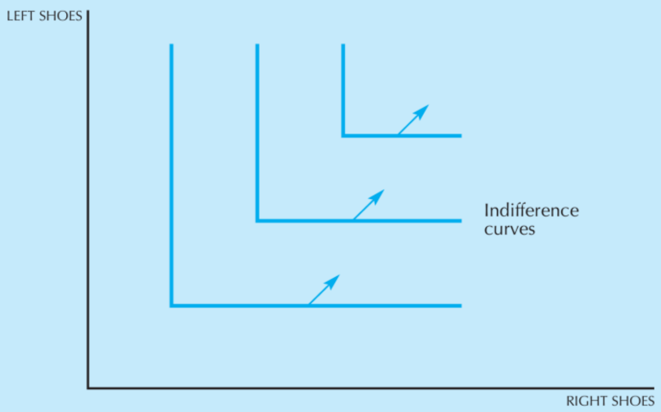


Various kinds of preferences. Panel A depicts convex preferences, panel B depicts nonconvex preferences, and panel C depicts "concave" preferences.

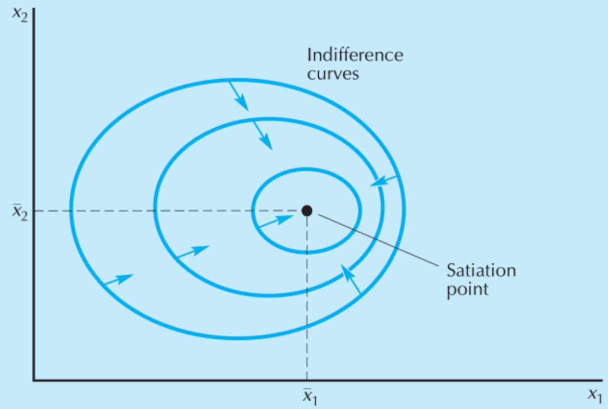
Indifference Curves: Other Examples



Perfect substitutes. The consumer only cares about the total number of pencils, not about their colors. Thus the indifference curves are straight lines with a slope of −1.



Perfect complements. The consumer always wants to consume the goods in fixed proportions to each other. Thus the indifference curves are L-shaped.



Satiated preferences. The bundle $(\overline{x}_1, \overline{x}_2)$ is the satiation point or bliss point, and the indifference curves surround this point.

Summary

1. Economists assume that a consumer can rank various consumption possibilities. The way in which the consumer ranks the consumption bundles describes the consumer's preferences.

2. Preference assumptions (Rational): Completeness, reflexivity, transitivity.

3. Indifference curves can be used to depict different kinds of preferences.

4. Well-behaved preferences are monotonic (meaning more is better) and convex (meaning averages are preferred to extremes)

Intermediate Microeconomics

Chapter 4: Utility

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Utility

Utility function is a way of describing "preferences".

It assigns numerical values to all bundles to allow ranking.

A utility function $U(x_1, x_2)$ represents a preference relation (if and only if):

$$(x_1, x_2) \succ (y_1, y_2) \quad \Leftrightarrow \quad u(x_1, x_2) > u(y_1, y_2)$$

$$(x_1, x_2) \prec (y_1, y_2) \quad \Leftrightarrow \quad u(x_1, x_2) > u(y_1, y_2)$$

$$(x_1, x_2) \sim (y_1, y_2) \quad \Leftrightarrow \quad u(x_1, x_2) = u(y_1, y_2)$$

Utility

If f(u) is any monotonic transformation of a utility function that represents some particular preferences, then $f(u(x_1,x_2))$ is also a utility function that represents those same preferences.

Why?

1. Particular preferences: $u(x_1, x_2) > u(y_1, y_2)$ if and only if $(x_1, x_2) \succ (y_1, y_2)$
2. If f(u) is a monotonic transformation, then $u(x_1, x_2) > u(y_1, y_2)$ if and only if $f(u(x_1,x_2)) > f(u(y_1,y_2))$
3. Therefore $f(u(x_1,x_2)) > f(u(y_1,y_2))$ if and only if $(x_1, x_2) \succ (y_1, y_2)$

Utility Functions: Cobb-Douglas

$$u(x_1, x_2) = A \cdot x_1^{c} \cdot x_2^{d} \qquad c > 0, d > 0$$

A convenient transformation of Cobb-Douglas:

$$u(x_1, x_2) = x_1^{\gamma}\, x_2^{1-\gamma} \qquad \gamma \in (0, 1)$$

How do Cobb-Douglas indifferent curves look like?

Utility Functions & Indiff. Curves: Cobb-Douglas

$$u(x_1, x_2) = x_1^{\gamma}\, x_2^{1-\gamma} \qquad \gamma \in (0,1)$$

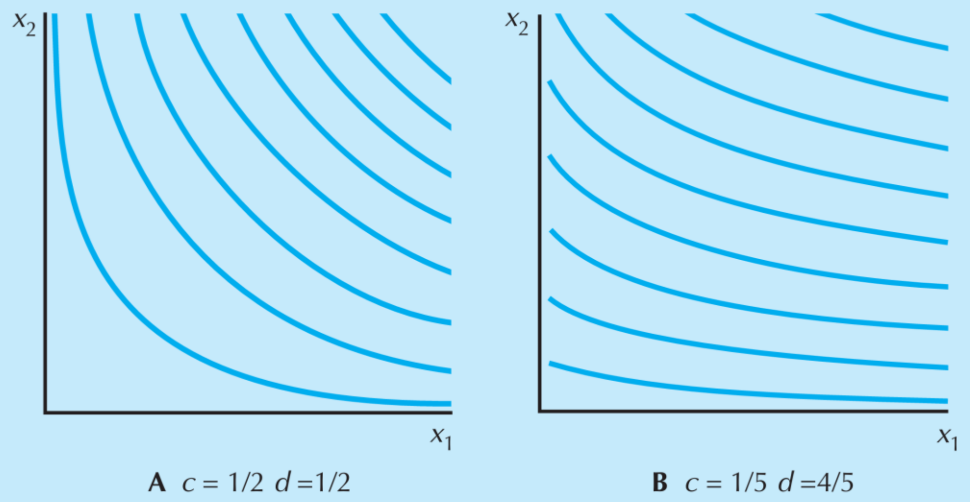
Indifferent curves of Cobb-Douglas:

$$u(x_1, x_2) = x_1^{\gamma}\, x_2^{1-\gamma} = \bar{u}$$

$$x_2 = \bar{u}^{\frac{1}{1-\gamma}} \cdot x_1^{\frac{-\gamma}{1-\gamma}}$$

Utility Functions & Indiff. Curves: Cobb-Douglas

Indifferent curves of Cobb-Douglas: $x_2 = \bar{u}^{\frac{1}{1-\gamma}} \cdot x_1^{\frac{-\gamma}{1-\gamma}}$ $\quad \gamma \in (0,1)$



Cobb-Douglas indifference curves. Panel A shows the case where $c = 1/2$, $d = 1/2$ and panel B shows the case where $c = 1/5$, $d = 4/5$.

Utility Functions: Others

Constant Elasticity of Substitution (CES):

$$u(x_1, x_2) = A \cdot (x_1^{\rho} + x_2^{\rho})^{\frac{1}{\rho}} \qquad \rho > 0$$

Perfect Substitute:

$$u(x_1, x_2) = \alpha x_1 + \beta x_2 \qquad \alpha > 0, \beta > 0$$

Perfect Complements:

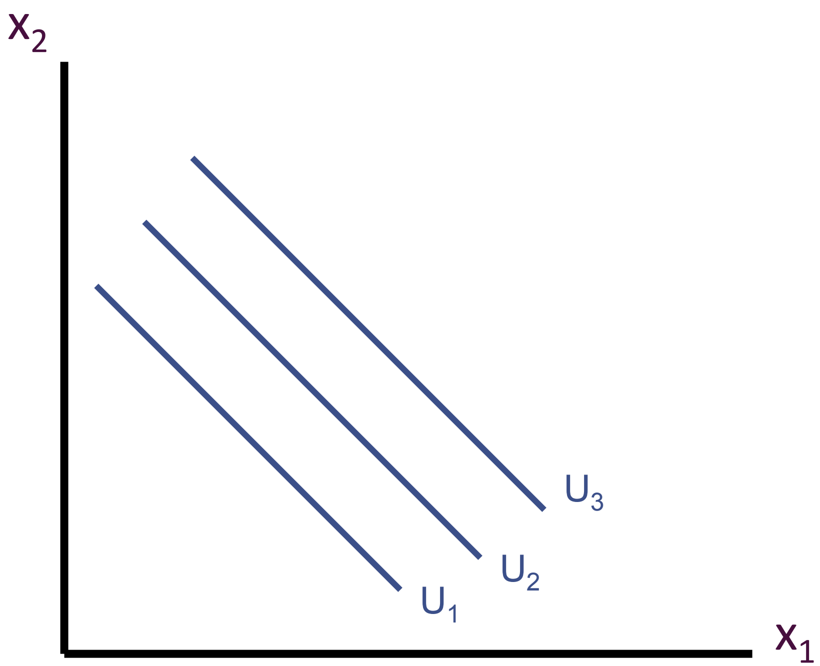
$$u(x_1, x_2) = \min(\alpha x_1, \beta x_2) \qquad \alpha > 0, \beta > 0$$

Quasilinear Preferences:

$$u(x_1, x_2) = v(x_1) + x_2 \qquad v'(x_1) > 0, v''(x_1) \leq 0$$

Utility Functions & Indiff. Curves: Perfect Substitute

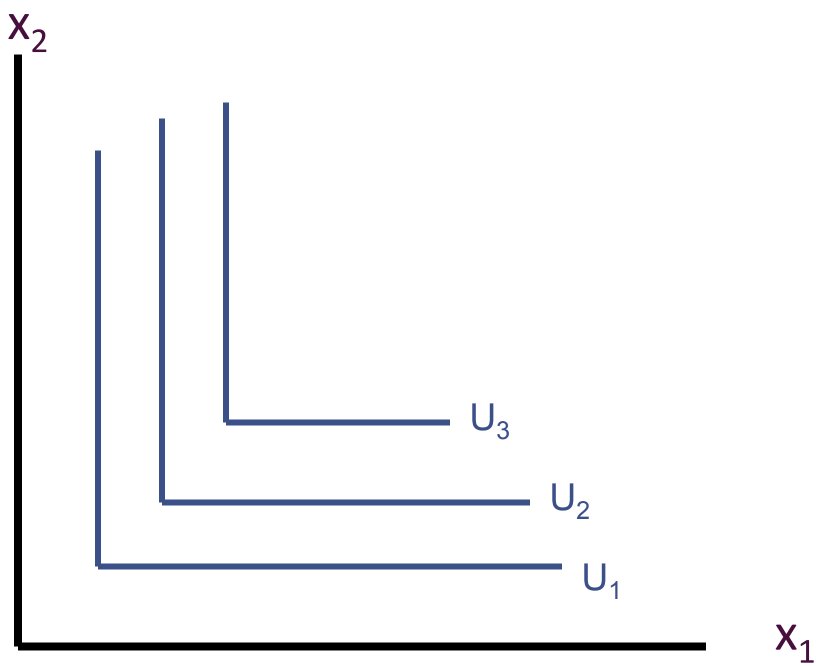
Indifferent curves of perfect substitute: $x_2 = \frac{\bar{u}}{\beta} - \frac{\alpha x_1}{\beta}$



Consumers are willing to trade at a constant rate, $\alpha : \beta$

All indifference curves have the same slope: $-\alpha/\beta$

Utility Functions & Indiff. Curves: Perfect Complements (Leontief)

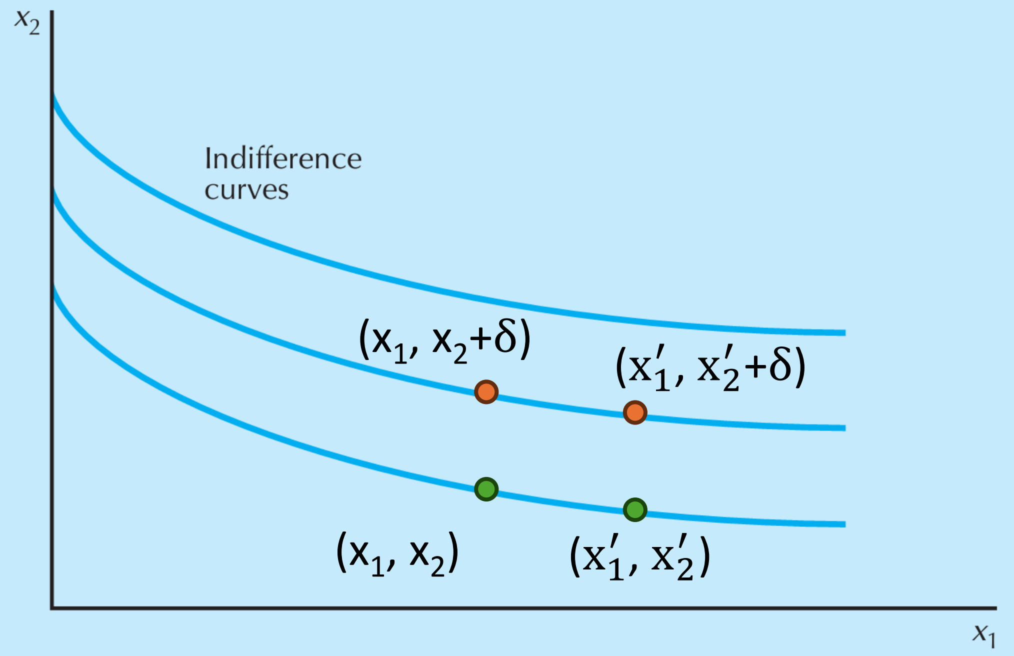


The minimum number of x_1, x_2 matters

All indifference curves are L-shaped

Utility Functions & Indiff. Curves: Quasilinear Preferences

Indifferent curves of quasilinear preferences: $x_2 = -v(x_1) + \bar{u}$



Quasilinear preferences. Each indifference curve is a vertically shifted version of a single indifference curve.

Each curve is a vertically shifted copy of the other

Marginal Utility

Marginal means “incremental”.

The marginal utility with respect to good 1 is the rate of change in total utility as the quantity of good 1 consumed change:

$$MU_1 = \frac{\partial u(x_1, x_2)}{\partial x_1} = \lim_{\Delta x_1 \to 0} \frac{u(x_1 + \Delta x_1, x_2) - u(x_1, x_2)}{\Delta x_1}$$

How to calculate the change in utility associated with a small change in consumption of good 1?

Marginal Utility and MRS

Marginal Rate of Substitution (MRS):

—The rate at which a consumer is just willing to substitute a small amount of good 2 for good 1, keeping utility constant.

—MRS measures the slope of the indifference curve at a given bundle of goods.

Derive the MRS

$$u(x_1, x_2) = \bar{u}$$

$$d\bar{u} = 0 = \frac{\partial u(x_1, x_2)}{\partial x_1} dx_1 + \frac{\partial u(x_1, x_2)}{\partial x_2} dx_2$$

$$MRS = \frac{dx_2}{dx_1} = -\frac{MU_1}{MU_2}$$

Marginal Utility and MRS: Cobb-Douglas

$u(x_1, x_2) = x_1^{\gamma}\, x_2^{1-\gamma} \qquad \gamma \in (0, 1)$

MRS= ?

Marginal Utility and MRS: Cobb-Douglas

$u(x_1, x_2) = x_1^{\gamma} x_2^{1-\gamma} \qquad \gamma \in (0, 1)$

$MRS = -\frac{MU_1}{MU_2} = -\frac{\partial u(x_1, x_2)}{\partial x_1} / \frac{\partial u(x_1, x_2)}{\partial x_2} = -\gamma x_2^{1-\gamma} x_1^{\gamma-1} / (1-\gamma) x_1^{\gamma} x_2^{-\gamma}$

$MRS = \frac{-\gamma}{1-\gamma} \frac{x_2}{x_1}$

If taking a logarithm, $u(x_1, x_2) = \gamma \ln x_1 + (1-\gamma) \ln x_2$

MRS=?

Marginal Utility and MRS: Others

Perfect Substitutes:

$$u(x_1, x_2)=\alpha x_1 + \beta x_2 \qquad \alpha > 0, \beta > 0$$

MRS=?

Perfect Complements:

$$u(x_1, x_2)= \min(\alpha x_1, \beta x_2) \qquad \alpha > 0, \beta > 0$$

MRS=?

Quasilinear Preferences:

$$u(x_1, x_2)=v(x_1) + x_2 \qquad v'(x_1) > 0, v''(x_1) \leq 0$$

MRS=?

Summary

1. A utility function is simply a way to represent or summarize preference ordering. The numerical magnitudes of utility levels have no intrinsic meaning.

2. Thus, given any one utility function, any monotonic transformation of it will represent the same preferences.

3. The marginal rate of substitution, MRS, can be calculated from the utility function via the formula $\mathrm{MRS} = \frac{\mathrm{dx}_2}{\mathrm{dx}_1} = -\frac{\mathrm{MU}_1}{\mathrm{MU}_2}$