幾道破題

Problem Sets for Macroeconomics

February, 2024

PS 1

Table 1.1: Data for PS 1.1 (Billions of dollars)

1	Personal consumption expenditures C	3500
2	Gross private domestic investment I	2800
3	Government consumption expenditures and gross investment G	960
4	Net exports of goods and services NX	320
5	Indirect business taxes less subsidies IBT-S	660
6	Current surplus of government enterprises (always domestic)	70
7	Business transfers, domestic	50
8	Compensation of employees, domestic	3200
9	Proprietors' income, domestic AI_1	350
10	Rental income of persons, domestic LI	450
11	Corporate profits, domestic AI_2	1600
12	Net Interest, domestic AI_3	900
13	Consumption of fixed capital (always domestic) D	300
* 14	Net factor payments (wages from the rest of the world)	120
15	Personal interest income	800
16	Personal dividend income	1200
17	Personal current transfer receipts from government and business	230
18	Contributions for government social insurance, domestic	340
19	Personal income taxes	1000

劳动收入 资本收入
GDP = Labor Income + Asset Income
间接税 - 补贴
+ (Indirect Business Taxes − Subsidies)
折旧
+ Depreciation

1.1 a. $GDP = C + I + G + NX = 3500 + 2800 + 960 + 320 = 7580$

$GDP = (3200 + 450) + (70 + 50 + 350 + 1600 + 900) + 660 + 300 = 7580$

b. $GNP = GDP + NFP = 7700$

$NNP = GNP - D = 7700 - 300 = 7400$

$NI = NNP - Dis = 7400$

c. $NNFI = 7400 - \underline{\underline{Net}}\ tax - 50 - 70$ 无法计算

$= 3200 + 350 + 450 + 1600 + 900 + 120 = 6620$

$GNFI = NNFI + D = 6920$

d. Net indirect tax = 560

$PI = 7400 - 560 - 70 - 50 + 230 - 340 - 1600 + 1200 - 900 + 800 = 6010$

$DPI = PI - 1000 = 5010$

本国要素收入
Net national factor income (NNFI on line 32)
= National income
− Net indirect business taxes
− Business transfer
− Surplus of government firms
= Compensation of employees
+ Proprietors' income
+ Rental income of persons
+ Corporate profits
+ Net interest and miscellaneous payments
Gross national factor income (GNFI on line 29)
= NNFI + Consumption of fixed capital.
国民折旧数

Personal income (PI) is the income that persons receive.
PI = NI − Indirect business taxes 间接税
− Current surplus of government enterprises 国有企业剩余
− Business transfer 转移支付
+ Personal current transfer receipts from gov and business
− Social insurance contribution (domestic)
− Corporate profits + Personal dividends income
− Net interest + Personal interest income
☆ PI = Personal current transfer receipts from gov and business
− Social insurance contribution (domestic)
+ Compensation of employees
+ Proprietors' income + Rental income of persons
+ Personal dividends income + Personal interest income.
Disposable Personal Income(DPI) is the amount households and noncorporate businesses have available to spend or save after satisfying their tax obligations to the government.
DPI = PI − Personal Current Taxes. 个人所得税

1.1 (数值计算题). The nation of Narnia has macroeconomic data, as shown in table 1.1. Suppose that there is no statistical discrepancy and that net factor payments include only compensation from the rest of the world.

a. Calculate Narnian GDP using the expenditure approach and the income approach.
b. Find gross national product, net national product, national income.
c. Find gross national factor income and net national factor income.
d. Find personal income and disposable personal income.

1 (PS1.1). a. GDP=7580; b. GNP=7700, NNP=NI=7400; c. GNFI=6920, NNFI=6620; d. PI=6010, DPI=5010.

a) i) GDP = Total value added $= 15\times5 - 25\times5 + 10\times20 = 150$

ii) $GDP = C + I + NX = 8\times20 + 2\times20 - 10\times5 = 150$

iii) $GDP = 50 + 40 + (15\times5 - 50) + (10\times20 - 25\times5 - 40) = 150$

b) $CA = 20\times2 - 10\times5 = -10$

c) $GNP = GDP + NFP = 150$

d) $GNP' = GDP + (15\times5 - 50) = 175$

✓ **1.2.** The nation of Narnia is comprised of a coal producer, a steel producer, and some consumers (there is no government). In a given year, the coal producer produces 15 million tons of coal and sells it for \$5 per ton. The coal producer pays \$50 million in wages to consumers. The steel producer uses 25 million tons of coal as an input into steel production, all purchased at \$5 per ton. Of this, 15 million tons of coal come from the domestic coal producer and 10 million tons are imported. The steel producer produces 10 million tons of steel and sells it for \$20 per ton. Domestic consumers buy 8 million tons of steel, and 2 million tons are exported. The steel producer pays consumers \$40 million in wages. All profits made by domestic producers are distributed to domestic consumers. There is no depreciation.

a. Determine Narnian GDP using (i) the product approach, (ii) the expenditure approach, and (iii) the income approach.
b. Determine Narnian current account surplus.
c. Find GNP of Narnia.
d. Determine GNP and GDP in the case where the coal producer is owned by foreigners, so that the profits of the domestic coal producer go to foreigners and are not distributed to domestic consumers.

2 (PS1.2). a. GDP=\$150 million; b. CA=−\$10 m; c. GNP=\$150 m; d. GNP = GDP + NFP = 150−25 = \$125 m.

1.3. The nation of Narnia is comprised of a corn producer, some consumers, and a government. The government does not make any investment. In a given year, the corn producer grows 30 million bushels of corn and the market price for corn is $5 per bushel. Of the 30 million bushels produced, 20 million are sold to consumers, 5 million are stored in inventory, and 5 million are sold to the government to feed the army. The corn producer pays $60 million in wages to consumers and $20 million in indirect taxes to the government. Consumers pay $10 million in income taxes to the government, receive $10 million in interest on the government debt, and receive $5 million in Social Security payments from the government. The corn producer is owned by consumers. Thus the profits of the corn producer are distributed to consumers. There is no depreciation.

a. Calculate GDP using (i) the product approach, (ii) the expenditure approach, and (iii) the income approach.

b. Calculate private disposable income, private sector saving, public sector saving, national saving, and the government deficit.

a) i) $GDP = 30 \times 5 = 150$ million \$.

ii) $GDP = C + I + G + NX$

$= 20 \times 5 + 5 \times 5 + 5 \times 5 = 150$

iii) $GDP = \underbrace{60}_{C_I} + \underbrace{(150-60-20)}_{P_I} + \underbrace{20}_{G_I} = 150$

b) $NI = NNP = GDP - \text{Depreciation} = 150$

$PI = NI - 20 + 15 = 145$

$DPI = PI - \text{Personal Current Taxes}$

$= 145 - 10 = 135$

$PI = NI -$ Indirect business taxes 间接税

− Current surplus of government enterprises 国有企业剩余

(− Business transfer 转移支付

\+ Personal current transfer receipts from gov and business) 个人实际得到

− Social insurance contribution (domestic) 社保分摊(国内)

(− Corporate profits + Personal dividends income) −分红 + 个人所得分红

(− Net interest + Personal interest income) 利息

$S^{Prv} = Y^d - C = DPI - C = 35$

$S^G = T - TR - INT^G - G^C = 30 - 5 - 10 - 5 = 10$

government deficit $= -S^G = -10$

$S = S^{Prv} + S^G = 45$

The Saving-Investment Identity 储蓄

Proposition 3 (The Saving-Investment Identity)

The current-account balance, denoted by CA, is the difference between total income (Y + NFP) and domestic total expenditure (C + I + G, also called domestic absorption). Then the following saving-investment identity holds: 国内吸收

经常账户

$CA = Y + NFP - (C + I + G)$

$= C + I + G + NX + NFP - (C + I + G)$

$= NX + NFP$

$$CA = S - \tilde{I}, \quad (2)$$

where S denotes the national saving which is the sum of private sector saving (S^{Prv}) [1] and public sector saving (S^G). $\tilde{I}$ denotes the national investment which is the sum of private investment (I) and public investment (G^I). $S - \tilde{I}$ represents net foreign investment (NFI, also called net capital outflow, NCO). For a closed economy, identity (2) implies $S = \tilde{I}$, that is, domestic saving is always equal to domestic investment. $S = S^{Prv} + S^G$

[1] S^{Prv} = Private disposable income (Y^d) − Private consumption (C). Y^d is the income of the private sector available to spend: $Y^d \triangleq GNP - (T - TR - INT^G)$ where TR the government transfer payments, INT^G the net interest payments from government, T the tax revenue. Y^d comprises disposable income of both households and businesses (ie., the private sector). If we assume $NFP = 0$, $TR = 0$, and $INT^G = 0$, then $Y^d = Y - T$.

1.4 (分析证明题). Suppose that the government does not make investment. Consider an identity $S^{\text{Prv}} - I = CA + D$, where S^{Prv} is private sector saving, I is gross private domestic investment, CA is the current account surplus, and D is the government deficit.

a. Show that the above identity holds.

b. Explain what the identity means.

$S^{Prv} = Y^d - C$

$= GNP - (T - TR - INT^G) - C$

$= Y + NFP - (T - TR - INT^G) - C = Y + NFP + D - C$

$= C + I + G^C + G^I + NX + NFP + D - C - G^C$

$= I + GI + CA + D$

$= I + CA + D$

故 $S^{Prv} - I = CA + D$

个人储蓄 = 经常账户盈余 + 个人投资 + 政府赤字

√ **1.5.** Suppose that government deficit is 10, interest on the government debt is 5, taxes are 40, government expenditures are 30, government gross investment is 0, personal consumption expenditures are 80, net factor payments are 10, current account surplus is −5, and national saving is 20. Calculate the following (not necessarily in the order given):

$D = 10 \quad INT^G = 5 \quad T = 40 \quad G = 30 \quad C = 80$

$NFP = 10 \quad CA = -5 \quad S = 20$

a. Government surplus (S^G);

b. Government consumption expenditures (G^C);

c. Transfers from the government to the private sector (TR);

d. Gross private domestic investment (I);

e. Net exports (NX);

f. Gross domestic product (GDP);

g. Gross national product (GNP);

h. Private disposable income (Y^d).

a) $S^G = -D = -10$

b) $G^C = 30 + 0 = 30$

c) $S^G = T - TR - INT^G - G^C = D - G^C$

$TR = 40 + 10 - 30 - 5 = 15$

d) $I = S - CA = 20 - (-5) = 25$

e) $NX = S - I - NFP = CA - NFP = -15$

f) $GDP = C + I + G + NX = 120$

g) $GNP = S + C + G = 130$

h) $Y^d = S^{Prv} + C = S + D + C = 110$

4 (PS1.4). a. Easy; b. Private saving, which is not used to finance domestic investment, is either lent to the domestic government to finance its deficit, or lent to foreigners.

5 (PS1.5). a. $S^G = -10$; b. $G^C = 30$; c. $TR = 15$; d. $I = 25$; e. $NX = -15$; f. $GDP = 120$; g. $GNP = 130$; h. $Y^d = 110$.

6 (PS1.6). a. 23.495828%; b. 36.846728%; c. 100.748599%, −15.815104%.

√ **1.6.** Chain-weighted growth rate of real GDP and quality improvement. In year 1 and year 2, there are two products produced in a given economy, computers and bread. Suppose that there are no intermediate goods. In year 1, 20 computers are produced and sold at $1,000 each, and in year 2, 25 computers are sold at $1,500 each. In year 1, 10,000 loaves of bread are sold for $1.00 each, and in year 2, 12,000 loaves of bread are sold for $1.10 each.

a. Calculate the percentage increase in real GDP from year 1 to year 2 using the chain-weighting method.

b. Calculate the percentage inflation rate from year 1 to year 2 using the chain-weighting method.

c. Suppose that computers in year 2 are twice as productive as computers in year 1. That is, computers are of higher quality in year 2 in the sense that one computer in year 2 is equivalent to two computers in year 1. How does this change your calculations in parts (a) and (b)?

year1

Product	Production	Price
computer	20	\$1000
bread	10000	\$1

year2

Product	Production	Price
computer	25	\$1500
bread	12000	\$1.1

a) $GDP_1 = 20 \times 1000 + 10000 \times 1 = 30000$

$GDP_2 = 25 \times 1500 + 12000 \times 1.1 = 50700$

$QI^L_{1,0} = \frac{\sum P_{i0} Q_{it}}{\sum P_{i0} Q_{i0}} = \frac{37000}{30000} = 1.2333$

$RGDP_2 = 20 \times 1500 + 10000 \times 1.1 = 41000$

$QI^P_{1,0} = \frac{\sum P_{it} Q_{it}}{\sum P_{it} Q_{i0}} = \frac{50700}{41000} = 1.236585$

$QI^F_{1,0} = \sqrt{QI^L_{t,0} \cdot QI^L_{t,1}} = 1.234958$

$a = 23.4958\%$

2

$$QI^L_{t,0} = \frac{\sum_i P_{i0} Q_{it}}{\sum_i P_{i0} Q_{i0}} \times 100 = \sum_i \left(\frac{P_{i0} Q_{i0}}{\sum_i P_{i0} Q_{i0}} \right) \frac{Q_{it}}{Q_{i0}} \times 100.$$

费雪物量指数

A **Fisher quantity index** for a pair of periods, $(t-1, t)$, is defined as

$$QI^F_{t,t-1} = \sqrt{QI^L_{t,t-1} \times QI^P_{t,t-1}}$$ L、P的几何平均

Employment (N) is the number of people who have a job. **Unemployment** (U) is the number of people who do not have a job but are looking for one. The **labor force** (L) is the sum of employment and unemployment.

$$L = N + U$$

$$\text{Unemployment Rate}(u) = \frac{\text{Unemployment}(U)}{\text{Labor Force}(L)} \times 100\%$$

$$\text{Participation Rate} = \frac{\text{Labor Force}(L)}{\text{Total Working Age Population}} \times 100\%$$

$$\text{Employment/Population ratio} = \frac{\text{Employment}(N)}{\text{Total Working Age Population}} \times 100\%$$

T: 100 m.
u: 5%.
U: 2.5 m

b) $u = \frac{U}{L} \times 100\%$ $L = 50$ m.
a) $PR = \frac{L}{T} \times 100\% = 50\%$ 3
c) $N = L - U = 47.5$ m
d) $R = \frac{N}{T} \times 100\% = 47.5\%$

1.7. Suppose that the unemployment rate is 5%, the total working-age population is 100 million, and the number of unemployed is 2.5 million. Determine (a) the participation rate; (b) the labor force; (c) the number of employed workers; and (d) the employment/population ratio.

√**1.8** (分析计算题)**.** House prices and bubbles. Houses can be thought of as assets with a fundamental value equal to the expected present discounted value of their future real rents.

a. Would you prefer to use real payments and real interest rates or nominal payments and nominal interest rates to value a house?

Suppose that the real rent R on a house received at the end of each period, the real interest rate r, and the real risk premium $\tilde{\alpha}$ are constant for ever. Let Q_t be the real price of a house at the beginning of period t. The rent on a house is like the dividend on a stock. Suppose Q_t is equal to the real fundamental value of a house.

b. Find the real price of a house.

c. If the interest rate falls, what will happen to the price-to-rent ratio?

d. If houses are perceived as a safer investment, what will happen to the price-to-rent ratio?

1.8 a) Real.
b) 由 $V(0) = \frac{\pi}{r}$，有 $P = \frac{R}{r+\tilde{\alpha}}$
c) ↑ d) ↑

$$Q_t = \sum_{i=0}^{\infty} \beta^{i+1} R = R(\beta + \beta^2 + \cdots) = R\frac{\beta}{1-\beta}, \quad \beta = \frac{1}{1+r+\tilde{\alpha}}$$
$$= \frac{R}{r+\tilde{\alpha}}$$
$$\frac{Q_t}{R} = \frac{1}{r+\tilde{\alpha}}$$

PS 2

a) $MPL = \frac{\partial Y}{\partial L} = \frac{1}{3}K^{\frac{1}{3}}H^{\frac{1}{3}}L^{-\frac{2}{3}}$ $H\uparrow$, $MPL\uparrow$

b) $MPH = \frac{\partial Y}{\partial H} = \frac{1}{3}K^{\frac{1}{3}}L^{\frac{1}{3}}H^{-\frac{2}{3}}$ $H\uparrow$, $MPH\downarrow$

c) 劳动份额 (α_L) $= \frac{(MPL)\cdot L}{Y} = \frac{\frac{1}{3}(K^{\frac{1}{3}}H^{\frac{1}{3}}L^{-\frac{2}{3}})\cdot L}{K^{\frac{1}{3}}H^{\frac{1}{3}}L^{\frac{1}{3}}} = \frac{1}{3}$

c. 产出的劳动份额是支付给劳动的产出所占的比例。支付给劳动的产出总量是实际工资（在完全竞争下，它等于劳动的边际产量）乘以劳动的数量。用这个数量除以产出总量就得到了劳动份额：

同理，人力资本份额 (α_H) $= \frac{1}{3}$
∴劳动份额 $= \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

d) $\frac{\text{Skilled Labor}}{\text{Unskilled Labor}}\xi = \frac{MPL + MPH}{MPL} = 1 + \frac{L}{H}$

$H\uparrow$，比值↓ $MPH<0$，提高了 UL 的 MP

注意，由于技能型工人的工资高于非技能型工人，这个比值总是大于 1。当 H 增加时，这个比值下降，原因是人力资本的边际报酬递减降低了其回报，而与此同时提高了非技能型工人的边际产量。

e) e. 更多的大学提供奖学金将会增加 H，这确实会导致一个更加平等的社会。额外的奖学金降低了教育的回报，缩小了受教育程度不同的工人之间的工资差距。更重要的是，由于技能型工人的数量增加时非技能型工人的边际产量上升，所以，额外的奖学金甚至提高了非技能型工人的绝对工资。

√**2.1.** Consider a Cobb-Douglas production function with three inputs. K is capital (the number of machines), L is labor (the number of workers), and H is human capital (the number of college degrees among the workers). The production function is $Y = K^{1/3}L^{1/3}H^{1/3}$.

a. Derive an expression for the marginal product of labor. How does an increase in the amount of human capital affect the marginal product of labor?

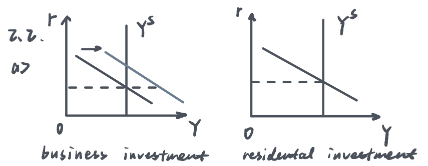
b. Derive an expression for the marginal product of human capital. How does an increase in the amount of human capital affect the marginal product of human capital?

c. What is the income share paid to labor? What is the income share paid to human capital? In the national income accounts of this economy, what share of total income would workers appear to receive?

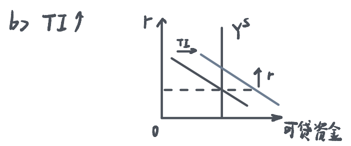
d. An unskilled worker earns the marginal product of labor, whereas a skilled worker earns the marginal product of labor plus the marginal product of human capital. Using your answers to parts (a) and (b), find the ratio of the skilled wage to the unskilled wage. How does an increase in the amount of human capital affect this ratio?

e. Some people advocate government funding of college scholarships as a way of creating a more egalitarian society. Others argue that scholarships help only those who are able to go to college. Do your answers to the preceding questions shed light on this debate?

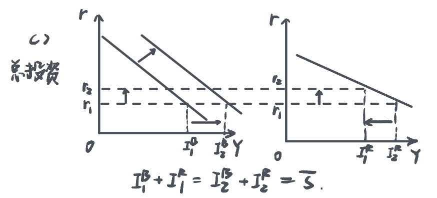
2.2. a)



b) TI↑

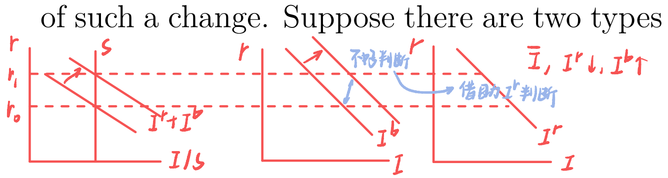


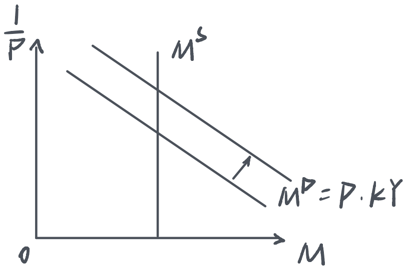
c) 总投资



$I_2^b + I_1^r = I_2^r + I_2^b = \bar{S}$.

2.2. A *tax credit* (免税额度) is a sum subtracted from the total amount that a taxpayer owes to the state (see Wiki's tax credit). When the government subsidizes investment by an investment tax credit, the subsidy often applies to only some types of investment. This question asks you to consider the effect of such a change. Suppose there are two types of investment in the economy:





$\uparrow Y = F(K, L, A)$, $\uparrow \frac{1}{P}$

business investment and residential investment. Public investment is zero. Suppose that the government institutes an investment tax credit only for business investment. According to the classical macroeconomic theory for a closed economy, answer the following questions.

a. How does this policy affect the demand curve for business investment? How about the demand curve for residential investment?
b. Draw the economy's supply of and demand for loanable funds. How does this policy affect the supply and demand for loanable funds? What happens to the equilibrium interest rate?
c. Compare the old and the new equilibria. How does this policy affect the total quantity of investment? How about the quantity of business investment? How about the quantity of residential investment?

✓ **2.3** (真实世界分析题). Some economic historians have noted that during the period of the gold standard, gold discoveries were most likely to occur after a long deflation. The discoveries of 1896 (Klondike Gold Rush) are such examples. Why might this be true?

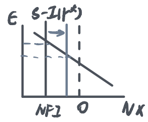
金本位

通货紧缩

O **2.4** (图形分析题). Let net factor payments (NFP) be fixed at 0. Suppose that there is no government investment. Suppose that the exchange rate floats freely. Suppose that China (Home) is a small open economy and the rest of the world (Foreign) can be regarded as a large open economy. With the aid of graphics, try to predict what would happen to China in response to each of the following events.

a. A fall in consumer confidence about Chinese future induces Chinese consumers to spend less and save more.
b. After financial crises, Foreign turns to protectionist trade policies to improve trade deficit.
c. Foreign begins to subsidize investment by instituting an investment tax credit.

China has become a large open economy recent years. What would happen to China in response to each of the above events.

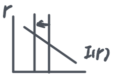


$\bar{r}$

a) $C\downarrow$, $S - I(r^*)\uparrow$. $\epsilon\downarrow$

$NX\uparrow$, $\bar{Y} = C + I + G + NX\uparrow$

b) $\bar{r}, \overline{NX}, \bar{Y}, \bar{C}$, $\epsilon\downarrow$ (?)

c) 

$r\uparrow$, $I\downarrow$, $\epsilon\downarrow$. $NX\uparrow$

a) $S\uparrow$, $r\downarrow$, $\epsilon\downarrow$, $NX\uparrow$, $\bar{Y}$, $C\downarrow$.

b) $\bar{r}$, $\overline{NFI}$, $\overline{NX}$, $\epsilon\downarrow$, $\bar{Y}$, $\bar{C}$

c) $r\uparrow$, $I\downarrow$ $\epsilon\downarrow$, $NX\uparrow$.

12 (PS2.4). Small Home: a. $\bar{r}$, ϵ depreciating, $NX\uparrow$, $\overline{Y}$, $C\downarrow$. b. $\bar{r}$, ϵ depreciating, $\overline{NX}$, $\overline{Y}$, $\overline{C}$. c. $r\uparrow$, $I\downarrow$, ϵ depreciating, $NX\uparrow$. Large Home: a. $r\downarrow$, ϵ depreciating, $NX\uparrow$, $\overline{Y}$, $C\downarrow$. b. $\bar{r}$, $\overline{NFI}$, $\overline{NX}$, ϵ depreciating, $\overline{Y}$, $\overline{C}$. c. $r\uparrow$, $I\downarrow$, ϵ depreciating, $NX\uparrow$.

O **2.5.** The world is made up of only two large countries: Eastland and Westland. Westland is running a large current account deficit and often appeals to Eastland for help in reducing this current account deficit. Currently, the government of Eastland purchases \$10 billion of goods and services, and all of these goods and services are produced in Eastland. The finance minister of Eastland proposes that the government purchase half of its goods from Westland. Specifically, the government of Eastland will continue to purchase \$10 billion of goods, but \$5 billion will be from Eastland and \$5 billion will be from Westland. The finance minister gives the following rationale: "Both countries produce identical goods so it does not really matter to us which country produced the goods we purchase. Moreover, this change in purchasing policy will help reduce Westland's large current account deficit." What are the effects of this change in purchasing policy on the current account balance in each country and on the world real interest rate?

$\bar{G} = G_E\uparrow + G_W\downarrow$ $\quad$ $G_E\uparrow$, $\text{Export}^{Priv}\uparrow$

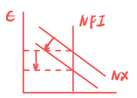
$CA = \bar{S} - \bar{I}$

$CA = \bar{S} - \bar{I}$ $\quad$ $CA = NX + NFP$

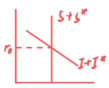
13 (PS2.5). NX and r^* are undisturbed.

E: $Y = C(Y-T) + I(r) + \overset{10}{G} + \overset{5}{NX(\epsilon)}$ $\quad$ $S = I(r) + NX(\epsilon)$

W: $Y = C(Y-T) + I(r) + G - \underset{-5}{NX(\epsilon)}$ $\quad$ $S^* = I(r^*) - NX(\epsilon)$



$\overline{NX}$, $\epsilon\downarrow$



O **2.6** (分析计算题). In a classical closed macroeconomic model, the demand for commodities is given by $Y^D = C(Y-T) + I(r) + G$, where Y is the income, T is the lump-sum tax, I is private investment, r is the real interest rate, and G is the government expenditure. The supply of commodities is

2.5 答：东部国家从西部国家购买 50 亿美元产品并没有改变总的政府购买支出量，因此对东部国家的国民储蓄无影响，两国经常账户余额不变。假定东部国家增加对西部国家产品的政府购买，则东部国家的私人部门将通过增加净出口抵消政府行为的影响。

在政府购买支出总量不变的情况下，东部国家政府购买政策的改变无法影响两国的经常账户余额和世界实际利率。因为经常账户等于储蓄减去投资，如果政府购买政策能够影响储蓄或投资，即政府购买支出总量发生变动，才能影响经常账户余额。

由$Y^S=Y^D$，有 $F(K,L,A)=C(Y-T)+I(r)+G$
对A求偏导，有 $F_A=\frac{\partial C}{\partial Y}F_A+I'(r)\frac{\partial r}{\partial A}$
$\underset{+}{\frac{\partial Y}{\partial A}=F_A}$ $\frac{\partial r}{\partial A}=\frac{1-C'}{I'}F_A$

由$M^S=M^D$，有 $PY=\frac{M}{k}$
对A求偏导，有 $\frac{\partial P}{\partial A}Y+PF_A=0$
$\Rightarrow \frac{\partial P}{\partial A}=-\frac{P}{Y}F_A$

14 (PS2.6). $\frac{\partial Y}{\partial A}=F_A$, $\frac{\partial r}{\partial A}=\frac{1-C'}{I'}F_A$, $\frac{\partial P}{\partial A}=-\frac{P}{Y}F_A$.

given by $Y^S=F(K,L,A)$, where F is a neoclasscial production function, K is the physical capital stock, L is the labor input, and A is the index of technology. In the money market, the demand for real money balance is given by $\frac{M^D}{P}=kY$, where $k>0$ is constant. The supply of nominal money is given by $M^S=M$. Suppose $C'(\cdot)\in(0,1)$ and $I'(r)<0$. Find the effects of an increase in A on Y,r, and P in equilibrium. (Calculate partial derivatives)

o **2.7.** Suppose Home is a small open economy and Foreign is a large open economy. Capital mobility is perfect. The classical open macroeconomic model can be written as the following.

$$Y^D=C(Y-T)+I(r)+G+NX(\epsilon),$$
$$Y^S=F(K,L,A),$$
$$r=r^*,$$

where Y is the income, T is the lump-sum tax, C is households' consumption, I is private investment, G is the government expenditure, NX is the net exports, ϵ is the real exchange rate, r is Home's real interest rate, r^* is Foreign's real interest rate, K is the physical capital stock in Home, L is the labor input in Home, and A is the index of technology in Home. F is Home's neoclassical production function. Suppose $C'(\cdot)\in(0,1)$, $I'(r)<0$, and $NX'(\cdot)<0$.

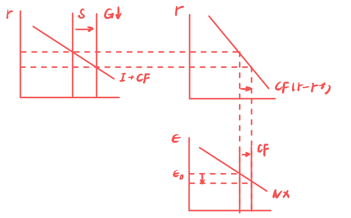
If a temporary adverse supply shock, captured by a temporary decrease of A, hits only the Home economy, what are the effects of the shock on Home's national saving, investment, and ϵ in equilibrium? (Calculate partial derivatives)

If capital mobility is imperfect, condition $r=r^*$ in the above is replaced by $CF(r-r^*)=NX(\varepsilon)$, where CF denotes net capital outflow and $-\infty<CF'<0$ (See Mankiw's appendix). What are the effects of the temporary adverse supply shock on Home's national saving, investment, and ϵ in equilibrium? (Calculate partial derivatives)

15 (PS2.7). Perfect capital mobility: $\frac{\partial Y}{\partial A}=F_A$, $\frac{\partial r}{\partial A}=0$, $\frac{\partial \epsilon}{\partial A}=\frac{1-C'}{NX'}F_A$, $\frac{\partial S}{\partial A}=(1-C')F_A$, $\frac{\partial I}{\partial A}=0$.
Imperfect capital mobility: Let CF denote net capital outflow. $\frac{\partial Y}{\partial A}=F_A$, $\frac{\partial r}{\partial A}=\frac{1-C'}{CF'+I'}F_A$, $\frac{\partial \epsilon}{\partial A}=\frac{1-C'}{NX'}\frac{CF'}{CF'+I'}F_A$, $\frac{\partial S}{\partial A}=(1-C')F_A$, $\frac{\partial I}{\partial A}=I'\frac{1-C'}{CF'+I'}F_A$.

$F(\bar{K},\bar{L},A)=C(Y-T)+I(r^*)+G+NX(\epsilon)$
$F_A=C_YF_A+NX'\frac{\partial \epsilon}{\partial A}$ $\frac{\partial \epsilon}{\partial A}=\frac{(1-C_Y)F_A}{NX'}<0.$
$S=Y-C-G$ $\frac{\partial S}{\partial A}=F_A-C_Y\cdot F_A=F_A(1-C_Y)>0$
$\frac{\partial I(r^*)}{\partial A}=0$

$Y=C(Y-T)+I(r)+G+NX(\epsilon)$
$\left.\begin{array}{l}\underbrace{S-I(r)}_{\triangleq CF(r)}=NX(\epsilon)\end{array}\right\}\Rightarrow CF(r-r^*)=NX(\epsilon)$ $CF'<0$
$\Rightarrow Y=C(Y-T)+I(r)+G+CF(r-r^*)$
$S=I(r)+CF(r-r^*)$

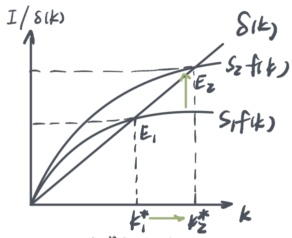


$\dot{K}=sY-\delta K.$
人均储蓄 $\frac{s\cdot Y}{L}\uparrow$ 人均产出 $y=\frac{Y}{L}\uparrow$

PS 3

索罗模型中节俭的美德。 $\Rightarrow A=0$

✓ **3.1.** The virtue of thrift in the Solow–Swan Model. Suppose there is no technological progress. If each person saves more in the Solow–Swan model, does the saving per worker rise? What is the effect of an increase in the saving rate on the output per worker?



开始时，储蓄率s_1和资本存量水平k_1^*时，投资与折旧抵消。
$s\uparrow$，$I\uparrow$，资本存量折旧δk_1^*，投资大于折旧。资本存量↑
直到经济达到新稳定状态k_2^*。新状态下有$k_2^*>k_1^*$，$s_2f(k_2^*)>s_1f(k_1^*)$

16 (PS3.1). The saving per worker rises. At the time when the saving rate rises, the output per worker does not change. But the output per worker is higher in the new steady state.

✓ **3.2.** Suppose that the economy's production function is $Y=\sqrt{K}\sqrt{AL}$, that the saving rate, s, is equal to 16%, and that the rate of depreciation, δ, is equal to 10%. Suppose further that the number of workers grows at 2% per year and that the rate of technological progress is 4% per year.

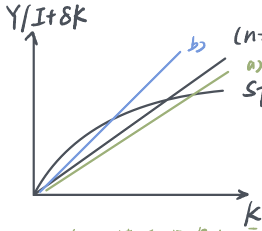
a. Find the steady-state values of the variables listed below.
  i. The capital stock per effective worker;
  ii. Output per effective worker;
  iii. The growth rate of output per effective worker;
  iv. The growth rate of output per worker;
  v. The growth rate of output.

17 (PS3.2). a. (i) $\hat{k}^*=1$; (ii) $\hat{y}^*=1$; (iii) 0; (iv) 4%; (v) 6%. b. (i) $\hat{k}^*=0.64$; (ii) $\hat{y}^*=0.8$; (iii) 0; (iv) 8%; (v) 10%. c. (i) $\hat{k}^*=0.64$; (ii) $\hat{y}^*=0.8$; (iii) 0; (iv) 4%; (v) 10%. People in (a) are better off.

$\frac{Y}{AL}=\left(\frac{K}{AL}\right)^{\frac{1}{2}}$ $\frac{f(\hat{k}^*)}{\hat{k}^*}=\frac{n+g+\delta}{s}$
$\hat{y}=\hat{k}^{\frac{1}{2}}=f(\hat{k})$ $\hat{k}^*=\left(\frac{s}{n+g+\delta}\right)^2$
均衡条件：$sf(\hat{k}^*)=g+n+\delta.$

$Y=\sqrt{K}\sqrt{AL}$. $s=16\%$ $\delta=10\%$
$n=2\%$ $g=4\%$.

a) i) 稳态时有 $\hat{k}^*=\frac{K}{AL}=1$
ii) 稳态时，有 $\dot{\hat{k}}^*=sf(\hat{k}^*)-(n+\delta+g)\hat{k}^*=0$
$\hat{k}^*=\frac{sf(\hat{k}^*)}{n+\delta+g}=\frac{16\%f(\hat{k}^*)}{16\%}=1$
$\hat{y}^*=f(\hat{k}^*)=1$
iii) 0 iv) $\alpha=g=4\%$ v) $\alpha=g+n=6\%$
b) i) $\hat{k}^*=1-\left(\frac{6}{10}\right)^2=0.64$
ii) $\hat{y}^*=\frac{\hat{k}^*(n+\delta+g)}{s}=0.8$
iii) 0 iv) $\alpha=g=8\%$ v) $\alpha=g+n=10\%$
c) i) $\hat{k}^*=1-\left(\frac{6}{10}\right)^2=0.64.$
ii) $\hat{y}^*=0.8$
iii) 0 iv) $\alpha=g=4\%$ v) $\alpha=g+n=10\%$
A更好 $\hat{k}_a^*>\hat{k}_c^*$.



a) s↓, 持平投资斜率 n+g+δ ↓.
k^*↑.

b) g↑, n+g+δ↑, k^*↓

(c)生产函数是柯布—道格拉斯型的 $f(k)=k^{\alpha}$，并且资本份额 α 上升的影响

由于持平投资线的斜率为 $(n+g+\delta)$，因此 α 上升对持平投资线没有影响。由于实际投资线为 $sf(k)$，而 $f(k)=k^{\alpha}$，因此 $\frac{\partial sk^{\alpha}}{\partial\alpha}=sk^{\alpha}\ln k$。当资本份额 α 上升时，实际投资线的变化需要分情况讨论：对于 $0<\alpha<1$，如果 $\ln k>0$，或者 $k>1$，$\partial sk^{\alpha}/\partial\alpha>0$，即实际投资线 sk^{α} 随 α 增加而上升，则新的实际投资线位于旧的实际投资线之上；反之，如果 $\ln k<0$，或者 $0<k<1$，$\partial sk^{\alpha}/\partial\alpha<0$，则新的实际投资线位于旧的实际投资线之下；对于 $k=1$，则新的实际投资线与旧的实际投资线重合。

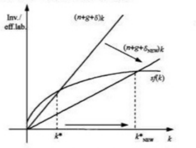


图 1–3　折旧率下降的影响

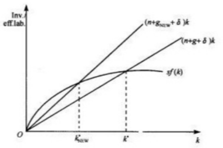


图 1–4　技术进步率上升对稳态人均资本存量的影响

除此之外，α 上升对于 k^* 的影响还受到 s 和 $(n+g+\delta)$ 的大小的影响。如果 $s>(n+g+\delta)$，α 的上升会使 k^* 上升，如图 1–5 所示。

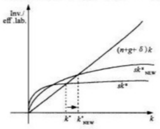


图 1–5　资本份额 α 上升的影响

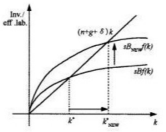


图 1–6　单位有效产出比以前更高的影响

(d)工人们发挥更大的努力，使得对于单位有效劳动的资本的既定值，单位有效劳动的产出比以前有更高的影响：

如果修改密集形式的生产函数形式为：$sBf(k)$，$B>0$，则实际投资线为 $sBf(k)$。

工人们更加努力的劳动，则单位有效劳动的产出比以前提高，即表现为 B 上升，B 的上升会使实际投资线 $sBf(k)$ 上升；持平投资线 $(n+g+\delta)k$ 并不受影响，此时，k 也从 k^* 上升到 k^*_{NEW}。如图 1–6 所示。

3.4. a. $Y=K^{\alpha}(AL)^{1-\alpha}$.　$\hat{y}=f(\hat{k})=\hat{k}^{\alpha}$

$\alpha=0.3$　$g+n=0.03$　$\delta=0.04$

$\frac{K}{Y}=2.5$

$sf(\hat{k}^*)=(n+g+\delta)\hat{k}^*$.

$\frac{Y}{K}=\frac{f(\hat{k}^*)}{\hat{k}^*}=\frac{n+g+\delta}{s}=\frac{1}{2.5}$

$\Rightarrow s=$

$f'(\hat{k}^*)=\alpha\hat{k}^{*\alpha-1}=\alpha\cdot\frac{\hat{k}^{*\alpha}}{\hat{k}^*}=\alpha\frac{f(\hat{k}^*)}{\hat{k}^*}$

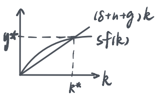
b. $\alpha\frac{f(\hat{k}_G)}{\hat{k}_G}=\alpha\hat{k}_G^{\alpha-1}=f'(\hat{k}_G)=n+g+\delta$

$s_G\cdot f(\hat{k}_G)=(n+g+\delta)\hat{k}_G$

$\frac{f(\hat{k}_G)}{\hat{k}_G}=\frac{n+g+\delta}{s_G}=\frac{n+g+\delta}{s}$

3.5 a. 总收入增长率 $\alpha_1=n+g$

故 $\alpha_{11}=\alpha_{12}$



b. $y_1^*=y_2^*$.

$y_1^*E_1>y_2^*E_2$.

故 $(\frac{Y}{L})_1>(\frac{Y}{L})_2$ 人均收入更高

c. $MPK=k_1^*=k_2^*=k^*$

d. $w=f(k)-MPK\cdot k$　$w_1=w_2$

$w_1E_1>w_2E_2$. 故人均有效工资更高.

20 (PS3.5). a. The same. b. Higher in Westland. c. The same. d. Higher in Westland.

3.6 $\frac{Y}{AL}=F(\frac{K}{AL},1)\Rightarrow\hat{y}=f(\hat{k})$.

$\hat{k}=\frac{K}{AL}\Rightarrow\dot{\hat{k}}=\frac{\dot{K}AL-K(\dot{A}L+A\dot{L})}{(AL)^2}$

$=\frac{\dot{K}}{AL}-\frac{K}{AL}\frac{\dot{L}}{L}-\frac{K}{AL}\cdot\frac{\dot{A}}{A}$

$=\frac{\dot{K}}{AL}-\hat{k}(n+g)$

$I=S=\frac{\partial F(K,AL)}{\partial K}K$, $I-\delta K=\dot{K}\Rightarrow\dot{K}=\frac{\partial F(K,AL)}{\partial K}\cdot K-\delta K$.

b. Suppose that the rate of technological progress doubles to 8% per year. Recompute the answers to part (a).

c. Now suppose that the rate of technological progress is still equal to 4% per year, but the number of workers now grows at 6% per year. Recompute the answers to part (a). Are people better off in part (a) or in part (c)?

✓ **3.3.** Describe how each of the following events affects the break-even and actual investment curves in the Solow–Swan model:

a. The rate of depreciation falls.

b. The rate of technological progress rises.

c. The production function is $Y=K^{\alpha}(AL)^{1-\alpha}$ and capital's share, α, rises.

d. Workers exert more effort, so that output per unit of effective labor for a given value of capital per unit of effective labor is higher than before.

✓ **3.4.** In the nation of Narnia, the capital share of GDP is about 30 percent, the average growth in output is about 3 percent per year, the depreciation rate is about 4 percent per year, and the capital-output ratio (K/Y) is about 2.5. Suppose that the production function is $Y=K^{\alpha}(AL)^{1-\alpha}$ and that the nation of Narnia has been in a steady state.

a. What must the saving rate be in the steady state? What is the marginal product of capital in the steady state?

b. Suppose that public policy raises the saving rate so that the economy reaches the Golden Rule level of capital. What must the saving rate be to reach the Golden Rule steady state? What will the marginal product of capital be at the Golden Rule steady state? What will the capital-output ratio be at the Golden Rule steady state?

W✓ **3.5.** The amount of education that a typical person receives varies substantially among countries. Suppose you were to compare Westland with a highly educated labor force and Eastland with a less educated labor force. Assume that the production function is given by $Y=F(K,EL)$, where education, denoted by E, affects only the level of the efficiency of labor L. Also assume that the two countries are otherwise the same: they have the same saving rate s, the same depreciation rate δ, the same population growth rate n, and the same growth rate g of education. Both countries are described by the Solow–Swan model and are in their steady states. Compare the following variables between two countries?

a. The rate of growth of total income.

b. The level of income per worker.

c. The real rental price of capital.

d. The real wage per worker.

54. 答：因为教育是影响劳动效率的一个重要因素，可以用 E 表示，所以教育水平会影响人均收入增长率。在本题中，国家 1 有比国家 2 受教育程度更高的劳动力，所以国家 1 中的每个工人都是更有效率的，即 $E_1>E_2$。

(1) 在索洛增长模型中，总收入的增长率为 $n+g$，这与劳动力的受教育水平无关。这样，这两个国家有相同的总收入增长率，因为它们有相同的人口增长率和技术进步率。

(2) 因为两个国家有相同的储蓄率，有相同的人口增长率和技术进步率，它们将达到同样的效率工人人均资本量 k^* 的稳态总水平。如图 3–2a–12 所示，因此稳态时效率工人的人均产出是 $y^*=f(k^*)$，这在两个国家中都是一样的。但是 $E_1>E_2$，所以 $y^*E_1>y^*E_2$，即 $(Y/L)_1>(Y/L)_2$，这样，劳动力受教育程度高的国家人均收入水平也高。

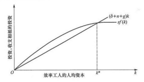


图 3–2a–12　效率工人的人均产出

(3) 因为 MPK 取决于效率工人的人均资本量，在稳定状态时，$k_1^*=k_2^*=k^*$。因为两个国家有相同的储蓄水平、相同的人口增长率和相同的技术进步率，所以 $R_1=R_2=MPK$，资本的实际租赁价格在两个国家中是相等的。

(4) 总产出由资本收入和劳动收入进行分割。因此工人人均效率工资可表示为：

$w=f(k)-MPK\times k$

正如 (2) 和 (3) 所讨论的，两个国家有相同的稳态资本存量 k 和 MPK，因此，两个国家人均效率工资是相等的。

然而，工人最关心的是人均工资而不是人均效率工资，而且我们实际能观测到的也是人均工资而不是人均效率工资。人均工资和人均效率工资可用下式表示：

人均工资 $=wE$

这样，劳动力受教育程度高的国家人均工资更高。

W✓ **3.6.** For a neoclassical production function $Y=F(K,AL)$, show that each competitive factor of production earns its marginal product in the Solow–Swan model. Show that if all capital income is saved and all labor income is consumed, then the economy reaches the golden rule level of capital accumulation.

21 (PS3.6). $s\hat{y}-(n+g+\delta)\hat{k}=\hat{k}[f'(\hat{k})-(n+g+\delta)]$.　$S=f'(\hat{k})\cdot k$.

WW✓ **3.7.** Piketty (2014, p.572) finds that the average rate of return on capital is about 4 or 5 percent per year while the growth rate of output will not exceed 2 percent per year in the long run. Piketty (2014, p.571) concludes that if the private rate of return on capital is greater than the rate of growth of output,

故 $\dot{\hat{k}}=F'_K\cdot\hat{k}-\hat{k}(n+g+\delta)$

$F'_K=f'(\hat{k})$

$\dot{\hat{k}}=[f'(\hat{k})-(n+g+\delta)]\hat{k}$

$\dot{\hat{k}}=sf(\hat{k})-(n+g+\delta)\hat{k}$

故 $sf(\hat{k})-(n+g+\delta)\hat{k}=[f'(\hat{k})-(n+g+\delta)]\hat{k}$

即 $s\hat{y}-(n+g+\delta)\hat{k}=[f'(\hat{k})-(n+g+\delta)]\hat{k}$

【答案】（1）由题设条件及新古典增长模型的相关假定可知，人均产出 y=Y/AL，人均资本占有量 k=K/AL。对 k=K/AL 两边求关于时间的导数可得：

$$\dot{k}=\frac{\dot{K}AL-K(\dot{A}L+A\dot{L})}{(AL)^2}=\frac{\dot{K}}{AL}-\frac{K}{AL}\frac{\dot{L}}{L}-\frac{K}{AL}\frac{\dot{A}}{A}=\frac{\dot{K}}{AL}-k(n+g)$$

储蓄 $S=\partial F(K,AL)/\partial K\times K$，经济均衡有 $I=S=\partial F(K,AL)/\partial K\times K$，由净投资等于资本增量，即 $I-\delta K=\dot{K}$，可以得到 $\dot{K}=\partial F(K,AL)/\partial K\times K-\delta K$。代入可得：

$\dot{k}=\frac{F'_K\cdot K-\delta K}{AL}-k(n+g)=F'_K\cdot k-k(n+g+\delta)$。再由 $F'_K=f'(k)$ 可以得到

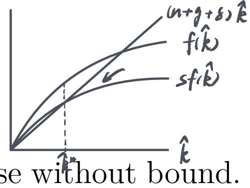
$\dot{k}=[f'(k)-(n+g+\delta)]\cdot k$。以下证明均衡增长路径的存在性、唯一性和稳定性。

因为新古典增长模型有所谓的稻田条件，即当资本量趋于零时资本的边际产出无穷大；当资本量很大时边际产出趋于零。

故当 $k=0$ 时，存在 k^* 使得 $f'(k)=n+g+\delta$，也即均衡的 k^* 存在。生产函数的单调性保证了 k^* 的唯一性。

仍由以上条件可知，当 $k>k^*$ 时，$f'(k)<f'(k^*)=n+g+\delta$，$\dot{k}<0$，即 k 随时间增加减少；当 $k<k^*$ 时，$f'(k)>f'(k^*)=n+g+\delta$，$\dot{k}>0$，即 k 随时间增加而增加；故 k^* 稳定，即存在一个稳定的平衡增长路径。（2）该均衡增长路径上的 k 等于黄金律水平的 k

先求解黄金律水平的 k，$\max c^*=y-sy^*=f(k^*)-(n+g+\delta)k^*$，对 k 求导，有 $\frac{\partial c^*}{\partial k^*}=f'(k^*)-(n+g+\delta)=0$，故 $f'(k^*)=n+g+\delta$。而（1）的解恰好满足此条件，故该均衡增长路径上的 k 等于黄金律水平的 k。



$r=f'(\hat{k})-\delta > g+n$

$f(\hat{k})>g+n+\delta$ $(\hat{k}<\hat{k}^*)$

$\frac{K}{Y}\uparrow=\frac{\hat{k}^*}{f(\hat{k}^*)}=\frac{s}{g+n+\delta}$ 不能无限增长

资本边际收益递减. K过大会收敛至 $\hat{k}^*$.

capital will grows faster than output, and K/Y will increase without bound. Workers who own nothing but labor will become more and more dominated by those who own capital stock. Is it possible for Piketty to be FALSE in the Solow model?

√ **3.8.** In the Solow model without technological progress, let L denote total working age population and let $\xi \in (0,1)$ denote the participation rate. ξL is the labor force in the Solow economy. Suppose the labor force is totally employed, which is consistent with the assumption of vertical labor supply curve. The growth rate of L is $n>0$. The saving rate is s. The depreciation rate is δ. Suppose the production function is neoclassical and can be written as $Y=F(K,\xi L)$. What are the effects of an increase in ξ on K, Y, and C in the steady state of the Solow model? (Calculate partial derivatives)

3.8 $Y=F(K,\xi L)=F(K,\tilde{L})$, $\tilde{L}=\xi L$.

$y=\frac{Y}{\tilde{L}} \Rightarrow y=f(k)$, $k=\frac{K}{\tilde{L}}$.

$\dot{K}=sY-\delta K$.

$\dot{k}=sf(k)-(\delta+n)k$ $\qquad \frac{f(k^*)}{k^*}=\frac{\delta+n}{s}$

$K=k^*\tilde{L}=k^*\xi L$ $\qquad k^*(\delta+n,s)$.

$\frac{\partial K}{\partial \xi}=k^*L=\frac{K^*}{\tilde{L}}L=\frac{K^*}{\xi}$.

Y, C 同理.

√ **3.9.** Suppose the production function is $Y=(AK)^\alpha L^{1-\alpha}$. The growth rates of A and L are g and n, respectively. The depreciation rate of K is δ. Show that the economy converges to a balanced growth path, and find the growth rates of Y and K on the balanced growth path.

3.9 $Y=(AK)^\alpha L^{1-\alpha}=K^\alpha(A^{\frac{\alpha}{1-\alpha}}L)^{1-\alpha}=K^\alpha(\tilde{A}L)^{1-\alpha}$

$\tilde{A}=A^{\frac{\alpha}{1-\alpha}}$.

$\hat{k}=\frac{K}{\tilde{A}L}$ 故 $\gamma_K=\gamma_{\tilde{A}}+\gamma_L=\frac{\alpha}{1-\alpha}g+n$.

? W **3.10.** Quality improvement and embodied technologcial progress. We consider a specific case to understand embodied technological progress. Suppose the price of computers is equal to 5000 CNY and will not change forever. Suppose the technology level of computers, A, is equal to 1 in year 1 and 2 in year 2, that is, $A(1)=1$ and $A(2)=2$. It means that computers produced in year 2 are twice as productive as computers produced in year 1. If a firm invests 5000 CNY in computers in year 1, the firm's contribution to physical capital stock is $A(1)\times 5000=5000$ CNY. But the contribution is $A(2)\times 5000=10000$ CNY if the investment takes place in year 2. Generally, the productivity of physical capital stock built in time t depends only on the state of technology at time t and is undisturbed by subsequent technological progress. This view of technologcial progress indicates that technologcial progress must be embodied in new physical capital stock before it can raise output. Now we consider embodied technologial progress in the Solow model. Suppose the production function is

$$Y=K^\alpha(AL)^{1-\alpha}, \quad \alpha\in(0,1).$$

The growth rates of technology, A, and the number of workers, L, are g and n, respectively. The depreciation rate of K is $\delta\in(0,1)$. The fundamental equation of the Solow model becomes

$$\dot{K}(t)=A(t)[sY(t)]-\delta K(t),$$

where $A(t)sY(t)$ represents new investment embodying technology at time t. Show that the Solow economy with embodied technology converges to a steady state. What are the growth rates of Y and K in steady state?

3.10. $Y=K^\alpha(AL)^{1-\alpha} \Rightarrow \frac{Y}{AL}=\left(\frac{K}{AL}\right)^\alpha$

有 $\hat{k}=\frac{K}{AL}$ 故 $\gamma_K=\gamma_A+\gamma_L$

$\hat{k}=\frac{K}{AL} \Rightarrow \dot{\hat{k}}=\frac{\dot{K}}{AL}-\hat{k}(g+n)$

$\dot{K}=AsY-\delta K$.

$\dot{\hat{k}}=s\frac{Y}{L}-\hat{k}(\delta+g+n)$

$\dot{\hat{k}}=sf(k)-\hat{k}(\delta+g+n)$ $\qquad A=\frac{n+g+\delta}{s}$

$\gamma_K=(1+\frac{1}{1-\alpha})g+n$ $\qquad \gamma_Y=\frac{g}{1-\alpha}+n$

$\dot{K}=s\cdot AY-\delta K.=s\tilde{Y}-\delta K$

$\tilde{Y}=AY=AK^\alpha(AL)^{1-\alpha}=K^\alpha(A^{\frac{1}{1-\alpha}+1}L)^{1-\alpha}$

$B=A^{\frac{2-\alpha}{1-\alpha}}$ $\qquad \gamma_B=\frac{2-\alpha}{1-\alpha}g$

$\tilde{Y}=K^\alpha(BL)^{1-\alpha}$.

$\hat{y}=\frac{\tilde{Y}}{BL}$ $\qquad \hat{k}=\frac{K}{BL}$

$\frac{\dot{\hat{k}}}{\hat{k}}=\frac{\dot{K}}{K}-\gamma_B-n$

$=s\frac{\tilde{Y}}{K}-(\delta+\gamma_B+n)$

$=s\frac{\hat{y}}{\hat{k}}-(\delta+\gamma_B+n)$

$\dot{\hat{k}}=s\hat{y}-(\delta+\gamma_B+n)\hat{k}$.

? W **3.11.** Alter the Solow–Swan growth model so that the production technology is given by $Y=AK$, where Y is output, K is capital, and A is technology which is constant. Thus, output is produced only with capital.

a. Show that it is possible for income per person to grow indefinitely.

b. Also show that an increase in the saving rate increases the growth rate of per capita income.

3.11 $Y=AK$

$\dot{K}=\text{saving}-(n+\delta)K$

$\frac{\dot{k}}{k}=\frac{\dot{K}}{K}-n=sA-(\delta+n)$

$\dot{k}=sAk-(\delta+n)k$

$s\uparrow, \gamma_Y\uparrow, \gamma_K\uparrow$.

26 (PS3.11). $\dot{y}/y=sA-(n+\delta)$. If $sA>n+\delta$, y grows indefinitely.

c. From parts (a) and (b), what are the differences between this model and the Solow–Swan model without technological progress?

3.12. Government expenditure financed by a lump-sum tax. In the Solow–Swan model without technological progress, the government purchases G units of consumption goods in each period. The government finances its purchases G through lump-sum taxes on workers. Let L denote the number of workers. Each worker pays a lump-sum tax $\tau > 0$. Suppose that the government budget is balanced each period, that is, $G = T = \tau L$. Consumers consume a constant fraction of disposable income—that is, $C = (1 - s)(Y - T)$, where $s \in (0, 1)$ is the saving rate.

a. Determine the steady state of capital per worker, k^*.

b. Show that there can be two steady states, one with high k^* and the other with low k^*.

c. Ignore the steady state with low k^* (it can be shown that this steady state is "unstable"). Determine the marginal effects of an increase in τ on capital per worker and on output per worker in the steady state. What are the marginal effects of $\tau \uparrow$ on the steady-state growth rates of aggregate output, aggregate private consumption, and aggregate investment?

3.12 a) 由 $G=T=\tau L$，有

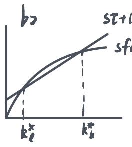
$\dot{K} = s(Y-G)+(1-\delta)K$

$= sY - s\tau L + (1-\delta)K$

$k \equiv \frac{K}{L}$　$k(1+n) = sf(k) - s\tau + (1-\delta)k$

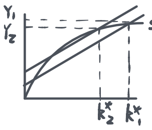
有 $sf(k^*) = s\tau + (n+\delta)k$

b)



c) $\tau\uparrow$, $k^*\downarrow$, $y^*\downarrow$

$\gamma_{Y^*} = \gamma_{C^*} = \gamma_{I^*} = n$ 与 τ 无关



$\dot{K} = \text{Saving} - \delta K$

$= Y - C - G - \delta K$

$= Y - (1-s)(Y-G) - G - \delta K$

$= s(Y - \tau L) - \delta K = sY - s\tau L - \delta K$

$\frac{\dot{K}}{K} = s\frac{Y}{K} - s\tau\frac{1}{k} - \delta$

$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - n = s\frac{y}{k} - s\tau\frac{1}{k} - \delta - n$

$\dot{k} = sy - s\tau - \delta - n = sf(k) - [s\tau + (\delta+n)k]$

$f(k^*) = \tau + \frac{\delta+n}{s}k^*$

$f'(k^*)\frac{\partial k^*}{\partial \tau} = 1 + \frac{\delta+n}{s}\cdot\frac{\partial k^*}{\partial \tau}$

$\frac{\partial k^*}{\partial \tau} = \frac{s}{sf'(k^*) - (\delta+n)} \begin{cases} <0, & k^*_{high} \\ >0, & k^*_{low} \end{cases}$

3.13. Government expenditure financed by a proportional tax. In the Solow–Swan model without technological progress, the production function is neoclassical: $Y(t) = F[K(t), L(t)]$, where Y, K, and L are the level of output, physical capital, and labor input, respectively. Suppose that the government always keeps a balanced budget constraint, that is, government expenditure $G(t)$ is equal to the tax revenue $T(t)$ for all t. Suppose that the tax revenue is a constant share of income $Y(t)$ for all t, that is, $T(t) = \tau \times Y(t)$ with a constant proportional tax rate $\tau \in (0, 1)$. The equilibrium in the market for goods is $Y(t) = C(t) + I(t) + G(t)$, where $C(t)$ is the aggregate private consumption and $I(t)$ is the aggregate private investment. Suppose that the consumption is $C(t) = c_1 \times [Y(t) - T(t)]$ where $c_1 \in (0, 1)$ is the constant marginal propensity to consume. Suppose that the government investment is a constant share of $G(t)$ for all t, that is, $G^I(t) = \zeta \times G(t)$ where $\zeta \in [0, 1]$. Let $n > 0$ be the constant growth rate of labor $L(t)$ and $\delta \in (0, 1)$ be the constant depreciation rate of physical capital stock $K(t)$. Find the marginal effect of an increase in τ on the physical capital per worker and the output per worker in the steady state.

3.13. $\dot{K} = \text{Saving} - \delta K = sY - \delta K$

$\text{Saving} = Y - C - G^C$

$= Y - c_1(Y-G) - (G - G^I)$

$= Y - c_1(Y - \tau Y) - (1-\zeta)\tau Y$

$= [1 - c_1(1-\tau) - (1-\zeta)\tau]Y$

$= sY$

$\dot{k} = sf(k) - (\delta+n)k \Rightarrow \frac{f(k^*)}{k^*} = \frac{\delta+n}{s}$

$\frac{\partial k^*}{\partial s} > 0$　$\frac{\partial k^*}{\partial s}\cdot\frac{\partial s}{\partial \tau} = c_1 + \zeta - 1 \begin{cases} >0 \\ =0 \\ <0 \end{cases}$

3.14. Suppose there are two countries, A and B, and each is a Solow–Swan economy without technological progress. In both countries, the growth rate of labor is $n > 0$ and the depreciation rate of physical capital is $\delta > 0$. In each country, a fraction a of the population is rich, and a fraction $1 - a$ is poor. Suppose that rich people save a fraction s_r of their income, and poor people save a fraction $s_p < s_r$ of their income, no matter what country they live in. In country A, suppose that rich people as a group receive a fraction χ_A of total income, while in country B rich people as a group receive χ_B fraction of total income. Assume $\chi_A > \chi_B$. If you were a poor person, where would you rather live, in country A or country B? What if you are rich?

3.14

	Rich	Poor
Popu.	a	$1-a$
saving	s_r >	s_p
χ	x	$1-x$

$\chi_A > \chi_B$

$y^P = \frac{(1-x)Y}{(1-a)L} = \frac{1-x}{1-a}y = \frac{1-x}{1-a}f(k^*)$

$\dot{K} = \text{Saving} - \delta K$

$= s_r x Y + s_p(1-x)Y - \delta K$

$= [s_r x + s_p(1-x)]Y - \delta K$

$= sY - \delta K$　$s \triangleq (s_r - s_p)x + s_p$.

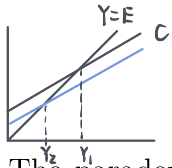
$\frac{\partial k^*}{\partial s} > 0$　$\frac{\partial s}{\partial x} = s_r - s_p > 0$

$\frac{\partial y^P}{\partial x} = \frac{-1}{1-a}f(k^*) + \frac{1-x}{1-a}f'(k^*) \times \frac{\partial k^*}{\partial s}\cdot\frac{\partial s}{\partial x} \gtreqless 0$

29 (PS 3.14). The rich would like to live in country A. The poor would like to live in A if $s_r/[(s_r - s_p)\chi + s_p] > 1/\alpha_K$.

$\frac{s_r}{(s_r - s_p)\chi + s_p} > \frac{1}{\alpha_K}$

PS 4

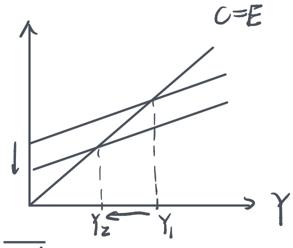


$C_0\downarrow$, $Y\downarrow$. 计划投资不变, 故 $S=I$.

4.1. The paradox of thrift in the Keynesian cross model. Suppose the consumption function is $C = C_0 + c_1 \cdot (Y - \overline{T})$. If households save more (captured by $C_0 \downarrow$) in the Keynesian cross model, what would happen to saving S? Should the virtues of thrift be recommended?

$I = b_0 + b_1 Y - b_2 i$

a) $C_0\downarrow$, $Y\downarrow$, $i\downarrow$.



$\frac{M^d}{P}$, $Y\downarrow$, 故 $i\downarrow$

?

4.2. The paradox of thrift in the IS-LM model. In the IS-LM framework, we assume that investment depends negatively on the interest rate and positively on output. That is, $I = b_0 + b_1 Y - b_2 i$. The consumption function is $C = C_0 + c_1 \cdot (Y - \overline{T})$. Suppose the demand for real money balance is $M^d/P = d_1 Y - d_2 i$. Suppose $b_0 > 0, b_1 > 0, b_2 > 0, C_0 > 0, c_1 > 0, d_1 > 0, d_2 > 0$, and $1 - c_1 - b_1 > 0$.

$\frac{I}{b_1} = \frac{b_0}{b_1} + Y - \frac{b_2}{b_1} i$ $\quad \frac{I}{b_1} - \frac{M^d}{Pd_1} = \frac{b_0}{b_1} + \left(\frac{d_2}{d_1} - \frac{b_2}{b_1}\right) i$

$\frac{M^d}{Pd_1} = Y - \frac{d_2}{d_1} i$ $\quad (b_1 d_2 - b_2 d_1) i$

a. Show the effect of the fall in consumer confidence (captured by $C_0 \downarrow$) on output and the interest rate.

b. How will the fall in consumer confidence affect consumption, investment, and private saving? Will the attempt to save more necessarily lead to more saving? $C_0\downarrow$, $C\downarrow$. $S=I$. 当 $b_1 d_2 - b_2 d_1 > 0$ 时, $I\downarrow$, $S\downarrow$.
<0 时, $I\uparrow$, $S\uparrow$.

4.3. Disturbances in the Mundell-Fleming (MF) model. Suppose that China, a small open economy, chooses fixed exchange rate regime and that the rest of the world, a large open economy, always chooses floating exchange rates regime. Net exports do not depend on income. Net factor payments are always 0. Use the MF model to predict effects of each of the following events on Chinese income, nominal exchange rate of CNY and Chinese net exports.

a. A fall in consumer confidence about Chinese future induces Chinese consumers to spend less and save more.

b. The introduction of a stylish line of Mercedes-Benz makes Chinese consumers prefer foreign cars over domestic cars.

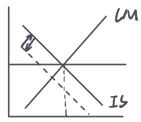
c. Introducing automated teller machines (ATM).

Suppose that China turns to floating exchange rate regime. Use the MF model to predict again effects of above events.

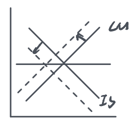
China has become a large open economy recent years. What would happen to China in response to each of the above events, under fixed and flexible exchange rates, respectively?

32 (PS4.3 ♡). Small China (flexible e): a. $\bar{i}, e\downarrow, \overline{Y}, NX\uparrow$; b. $\bar{i}, e\downarrow, \overline{Y}, \overline{NX}$; c. $\bar{i}, e\downarrow, Y\uparrow, NX\uparrow$. Small China (fixed e): a. $\bar{i}, \bar{e}, Y\downarrow, \overline{NX}(\bar{e}), M\downarrow$; b. $\bar{i}, \bar{e}, Y\downarrow, NX\downarrow, M\downarrow$; c. $\bar{i}, \bar{e}, \overline{Y}, \overline{NX}, M\downarrow$.

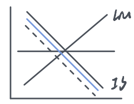
Large China (flexible e): a. $i\downarrow, e\downarrow, Y\downarrow, NX\uparrow, Y^*\downarrow$; b. $\bar{i}, e\downarrow, \overline{Y}, \overline{NX}, \overline{Y}^*$; c. $i\downarrow, e\downarrow, Y\uparrow, NX\uparrow, Y^*\downarrow$. Large China (fixed e): a. $\bar{i}, \bar{e}, Y\downarrow, \overline{NX}, M\downarrow, \overline{Y^*}$; b. $i\uparrow, \bar{e}, Y\downarrow, NX\downarrow, M\downarrow, Y^*\uparrow$; c. $\bar{i}, \bar{e}, \overline{Y}, \overline{NX}, M\downarrow, \overline{Y}^*$.



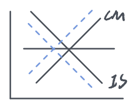
IS↓, e↓, IS↑, $\overline{Y}$, NX↑



$\bar{e}$, Y↓, $\overline{NX}$

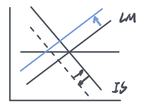


IS↓, e↓ Y↓, NX↑, Y*↓



$\bar{i}$, Y↓, $\bar{e}$, M↓, $\overline{Y^*}$, $\overline{NX}$

$\bar{e}$, Y↓, NX



$NX(\epsilon, \bar{i})$ $\quad \overline{Y} = \overline{C} + \overline{I} + \overline{G} + \overline{NX}$

IS↓, e↓, $\overline{Y}$, $\overline{NX}$

4.4. In years 2014 and 2015, Jilin province experienced a recession relative to the nation. Suppose that provinces are allowed to print CNY, and that Jilin is a small economy relative to the nation.

a. If Jilin province would try to improve its employment, would you recommend monetary expansion or fiscal expansion? Give your explanation.

b. If Jilin province turned to local protectionism which prohibited local firms from buying goods produced by other provinces, what would happen to the output and trade balance of Jilin province?

4.5. The Mundell-Fleming (MF) model takes the world interest rate i^* as an exogenous variable. Let's consider what happens when this variable changes.

a. What might cause the world interest rate to rise? (Hint: The world is a closed economy.)

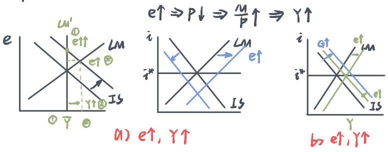
b. In the Mundell-Fleming model with floating exchange rates, what happens to aggregate income Y, the exchange rate e, and the trade balance $NX(eP/P^*)$ when the world interest rate rises?

c. In the Mundell-Fleming model with a fixed exchange rate, what happens to aggregate income Y, the nominal exchange rate e, and the trade balance $NX(eP/P^*)$ when the world interest rate rises?

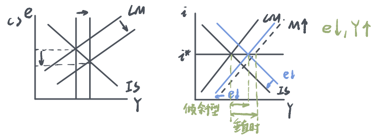
4.6. Suppose that the demand for real money balances depends on disposable income $Y - T$, so that the equation for the money market becomes $M/P = L(i, Y - T)$. Analyze the short-run impact of a tax cut in a small open economy on the exchange rate and income under floating and fixed exchange rates, respectively.

$\frac{M}{P} = L(i, Y)$ $\quad P = \lambda P_D + (1-\lambda)\frac{P_F}{e}$

$e\uparrow \Rightarrow P\downarrow \Rightarrow \frac{M}{P}\uparrow \Rightarrow Y\uparrow$



36 (PS4.7). a. Upward-sloping LM. b. $e\uparrow$, $Y\uparrow$. c. $e\downarrow$, $Y\uparrow$.



4.7. Suppose Home is a small open economy. Suppose that the price level relevant for money demand includes the price of imported goods and that the price of imported goods depends on the exchange rate. That is, the money market is described by $M/P = L(i, Y)$ and $P = \lambda P_D + (1-\lambda)P_F/e$ where P_D denotes the price of domestic goods, P_F the price of foreign goods measured in the foreign currency, e the nominal exchange rate, λ the share of domestic goods in the price index P. Assume that P_D and P_F are sticky in the short run.

a. Draw Home's LM curve for given values of P_D and P_F in the space (Y, e). Is it still vertical?
b. What is the effect of Home's expansionary fiscal policy under floating exchange rates?
c. What is the effect of Home's expansionary monetary policy under floating exchange rates?

37 (PS 4.8). $\frac{\partial Y}{\partial \pi^e} = \frac{1}{\Delta}[L_r(I' + CF')] > 0$. $\frac{\partial r}{\partial \pi^e} = \frac{1}{\Delta}[L_r(1-C')] < 0$. $\frac{\partial \epsilon}{\partial \pi^e} = \frac{CF'}{NX'}\frac{1}{\Delta}[L_r(1-C')] < 0$. $\Delta = -L_r(1-C') - (I' + CF')L_Y > 0$.

$Y = C(Y-T) + I(r) + G + NX(\epsilon)$ ①

$\frac{M}{P} = L(Y, r+\pi^e)$ 3个变量 Y, r, ϵ

$NX(\epsilon) = CF(r - r^*)$ ②

由①②, $Y = C(Y-T) + I(r) + G + CF(r-r^*)$ 两个变量 Y, r.

全微分 $(1-C')dY + (-I'-CF')dr = 0$

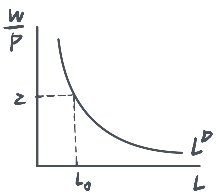
$L_Y dY + L_r dr + L_r d\pi^e = 0$ 即 $\begin{pmatrix} 1-C' & -I'-CF' \\ L_Y & L_r \end{pmatrix}\begin{pmatrix} dY \\ dr \end{pmatrix} = \begin{pmatrix} 0 \\ -L_r d\pi^e \end{pmatrix}$

$\Delta = (1-C')L_r - L_Y(I'+CF') > 0$

$dY = \frac{1}{\Delta}\begin{vmatrix} 0 & -I'-CF' \\ -L_r d\pi^e & L_r \end{vmatrix} = \frac{1}{\Delta}(I'+CF')L_r d\pi^e > 0$

4.8. In a small open economy with imperfect capital mobility and floating nominal exchange rates, the IS relation is given by $Y = C(Y - T) + I(r) + G + NX(\epsilon)$, where T is the lump-sum tax, r is the real interest rate, ϵ is the real exchange rate, $C'(\cdot) \in (0, 1)$, $I' < 0$, $NX'(\cdot) < 0$. The LM relation is given by $M/P = L(Y, r + \pi^e)$, where π^e is the expected rate of domestic inflation, $L_Y > 0$, $L_r < 0$. Suppose that there is no government intervention. The fundamental balance of payments identity without statistical discrepancy implies $NX(\epsilon) = CF(r - r^*)$, where r^* denotes the foreign real interest rate and CF denotes the net capital outflow or net foreign investment. Imperfect capital mobility in this model implies $CF'(\cdot) \in (-\infty, 0)$, contrasting with perfect capital mobility $[CF'(\cdot) = -\infty]$ in the Mundell-Fleming model (See Mankiw's appendix). What are the effects of an increase in π^e on the equilibrium levels of Y, r, and ϵ? (Calculate partial derivatives)

PS 5



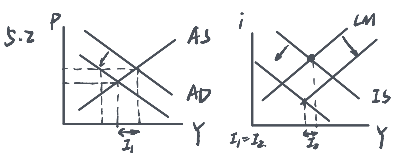
$Y = 10\sqrt{L}$

$\pi = P\cdot Y - RK - WL$

$\Rightarrow \frac{W}{P} = \frac{5}{\sqrt{L}} \Rightarrow L^D = \left(\frac{5}{W/P}\right)^2$

5.1. The sticky-wage model. Suppose the production function is $Y = 10\sqrt{L}$.

a. Find the demand for labor.
b. Suppose that the nominal wage rate is $W = 2$; the level of price and the expected level of price are equal to 1. Find the potential level of output.
c. In the sticky-wage model, the nominal wage rate is fixed at $W = 2$ by contract in the short run. Find the AS relation. $Y = 25P$

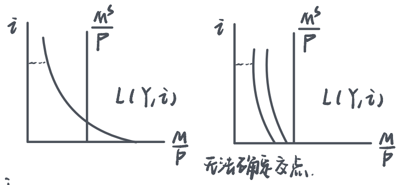


$C_0\downarrow, i\downarrow, S\uparrow.$

5.2. The paradox of thrift in the AS-AD model. Suppose the economy begins with output equal to its natural level. Then there is a decrease in consumer confidence (captured by $C_0\downarrow$) as households attempt to increase their saving for a given level of disposable income.

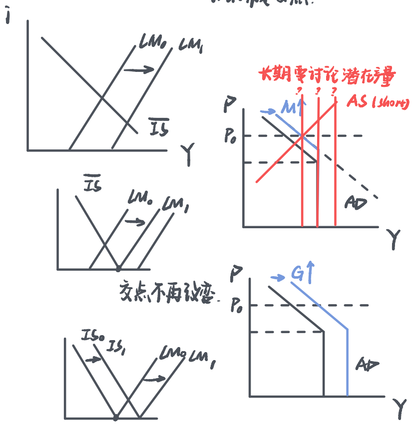
39 (PS5.2). a. In the short run, $Y\downarrow, i\downarrow, P\downarrow, C\downarrow$; for $I(i-\pi^e)$, investment and private saving rise; for $I(i-\pi^e, \underset{+}{Y})$, investment and private saving are ambiguous and the paradox may occur. b. In the long run, $Y = Y_n$. $i\downarrow, P\downarrow, C\downarrow$; investment and private saving rise. The paradox disappears.

40 (PS5.3). See the liquidity trap.



a. What happens to output, the interest rate, the price level, consumption, investment, and private saving in the short run? Is it possible that the decline in consumer confidence will actually lead to a fall in private saving in the short run?

b. Is there any paradox of saving in the long run?



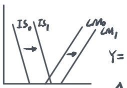
△ **5.3.** Suppose the money demand curve is flat when the interest rate i is equal to zero and downward-sloping as usual when the interest rate i is greater than zero.

a. Draw the IS, LM and AD curves.

Suppose that the economy starts from a short-run equilibrium where output is below the natural level of output. Based on your answer to part (a), answer the following questions.

b. If the central bank increases the money stock, what will be the effects on output and price level in the short run and the long run?

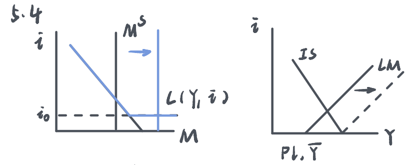
c. If the government raises government purchases, what will be the effects on output and price level in the short run and the long run?



$Y = C(Y-T, A) + I(r) + G$. $P\downarrow, A\uparrow, C\uparrow, Y\uparrow$

$P\downarrow \Rightarrow A\uparrow \quad \frac{\partial C}{\partial A} > 0 \Rightarrow C\uparrow \quad \frac{\partial Y}{\partial C} > 0 \Rightarrow Y\uparrow$

5.4



$P\downarrow, \bar{Y}$

△ **5.4.** Pigou Effect. Pigou (1943) said, falling prices would raise real wealth, which would also raise consumption expenditure. Thus the economy will not get stuck in the liquidity trap and automatically return to full employment. Suppose that the consumption is $C = C(Y - T, A)$, where $Y - T$ is the disposable income and A is the real wealth. Suppose $0 < \partial C/\partial Y < 1$ and $\partial C/\partial A > 0$. The nominal wealth includes bonds (B) and money (M). The real wealth is given by $A = (B + M)/P$. Is it possible for Pigou to be right?

41 (PS5.4 ♡). It is possible.

$$\begin{cases} IS: & Y = C(Y-T, \frac{B+M}{P}) + I(i-\pi^e) + G \\ LM: & \frac{M}{P} = L(Y, i) \end{cases}$$

$$\frac{\partial i}{\partial L}\begin{cases} <0, & i>0 \\ =0, & i=0 \end{cases}$$

$$\Rightarrow L_i(Y, 0) = \frac{\partial L}{\partial i} = -\infty$$

$$\begin{cases} (1-C_Y)dY - I'di + C_A\frac{B+M}{P^2}dP = 0 \\ -L_Y dY - L_i di - \frac{M}{P^2}dP = 0 \end{cases}$$

$$\begin{pmatrix} 1-C_Y & -I' \\ -L_Y & -L_i \end{pmatrix}\begin{pmatrix} dY \\ di \end{pmatrix} = \begin{pmatrix} -C_A\frac{B+M}{P^2}dP \\ \frac{M}{P^2}dP \end{pmatrix}$$

$$\Delta = \begin{vmatrix} 1-C_Y & -I' \\ -L_Y & -L_i \end{vmatrix} = \underbrace{-L_i(1-C_Y)}_{+} - L_Y I' > 0$$

$$dY = \frac{1}{\Delta}\begin{vmatrix} -C_A\frac{B+M}{P^2}dP & -I' \\ \frac{M}{P^2}dP & -L_i \end{vmatrix} = \frac{1}{\Delta}\left[C_A\frac{B+M}{P^2}dP L_i + I'\frac{M}{P^2}dP\right]$$

$$= \left[\frac{C_A(B+M)L_i + I'M}{\Delta}\frac{1}{P^2}\right]dP$$

$P\downarrow, Y\uparrow$ $\quad \frac{\partial Y}{\partial P} = \frac{C_A(B+M)L_i + I'M}{-L_i(1-C_Y) - L_Y I'}\cdot\frac{1}{P^2} = \frac{C_A(B+M) + \frac{I'M}{L_i}}{-(1-C_Y) - \frac{L_Y I'}{L_i}}\cdot\frac{1}{P^2} \overset{(L_i\to-\infty)}{=} \frac{C_A(B+M)}{-(1-C_Y)}\cdot\frac{1}{P^2}$

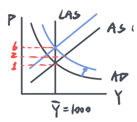
ⱱ **5.5.** Suppose that the AD function is $Y = 700 + 0.5\frac{M}{P}$, the AS function is $Y = 600(P - P^e) + 1000$, where Y is the level of output, P the actual price level, P^e the expected price level, and M the money supply. Let P and Y be positive. Suppose that an economy starts from $M = 600$ and $P^e = P$. Consider the effect of monetary expansion from $M^S = 600$ to $M^S = 3600$.

a. If the central bank makes an announcement of monetary expansion, find Y and P in the short-run equilibrium.

b. If the central bank increases the money supply secretly such that people do not know the monetary expansion, find Y and P in the short-run equilibrium.

a)
AD: $Y = 700 + 0.5\frac{M}{P}$

AS: $Y = 600(P - P^e) + 1000$



$$\begin{cases} 1000 = 700 + \frac{1}{2}\times\frac{600}{P} \\ 600 = \frac{600}{P} \end{cases} \Rightarrow P^e = P = 1$$

b)
$$\begin{cases} Y = 600(P-1) + 1000 \\ Y = 700 + \frac{1}{2}\times\frac{3600}{P} \end{cases} \Rightarrow \begin{cases} Y = 1600 \\ P = 2 \text{ (舍去负数)} \end{cases}$$

ⱱ **5.6.** A central bank has decided to adopt inflation targeting and is now debating whether to target 5 percent inflation or zero inflation. The economy is described by the following Phillips curve: $u = 5 - 0.5(\pi - \pi^e)$, where u and π are the unemployment rate and inflation rate measured in percentage points (i.e., $u = 5$ means the unemployment rate is 5%). The social cost of unemployment and inflation is described by the following loss function: $L = u + 0.05\pi^2$. The central bank would like this loss to be as small as possible.

a. If the central bank commits to target 5 percent inflation, what is expected inflation? If the central bank follows through, what is the unemployment rate? What is the loss from inflation and unemployment?

b. If the central bank commits to target zero inflation, what is expected inflation? If the central bank follows through, what is the unemployment rate? What is the loss from inflation and unemployment?

a) $u = 5 - 0.5(\pi - \pi^e)$

$L = u + 0.05\pi^2$

$L = u + 0.05\pi^2 = 5 - 0.5(\pi - \pi^e) + 0.05\pi^2$

Optional discretion.

$\frac{\partial L}{\partial \pi} = -0.5 + 0.1\pi = 0 \Rightarrow \pi_b = 5 \quad L = 6.25$

Optional rule. $(\pi - \pi^e = 0)$

$\frac{\partial L}{\partial \pi} = \frac{\partial}{\partial \pi}(5 + 0.05\pi^2) = 0 \Rightarrow \pi_R = 0 \quad L = 5$

c) choose optional rule $L_b < L_a$.

d) $\pi^e = 0 \quad L = u + 0.05\pi^2 = 5 - 0.5\pi + 0.05\pi^2$

$L = 3.75$

c. Based on your answers to parts (a) and (b), which inflation target would you recommend? Why? (b) L_b

d. Suppose the central bank chooses to target zero inflation, and expected inflation is zero. Suddenly, however, the central bank surprises people with 5 percent inflation. What is unemployment in this period of unexpected inflation? What is the loss from inflation and unemployment?

e. What problem does your answer to part (d) illustrate?

通胀目标和利率规则

5.7. Inflation targeting and the interest rate rule in the IS-LM model. Consider a closed economy in which the central bank follows an interest rate rule. The IS relation is given by $Y = C(Y - T) + I(Y, r) + G$, where r is the real interest rate, $C' \in (0, 1)$, $I_Y > 0$, $I_r < 0$, and $I_Y < 1 - C'$. The central bank sets the nominal interest rate i according to the rule

$$i = i^* + a(\pi^e - \pi^*) + b(Y - Y_n) \tag{5.1}$$

where π^e is expected inflation, π^* is the target rate of inflation, and Y_n is the natural level of output. Assume that $a > 1$ and $b > 0$. The symbol i^* is the target interest rate the central bank chooses when expected inflation equals the target rate and output equals the natural level. The central bank will increase the nominal interest rate when expected inflation rises above the target, or when output rises above the natural level. The rule (5.1) is slightly different from the Taylor rule in that the rule (5.1) uses expected inflation instead of actual inflation. Real and nominal interest rates are related by the Fisher equation $r = i - \pi^e$.

a. Let $r^* = i^* - \pi^*$. Use the definition of the real interest rate to express the interest rate rule (5.1) as

$$r = r^* + (a - 1)(\pi^e - \pi^*) + b(Y - Y_n). \tag{5.2}$$

b. Graph the IS relation in a diagram, with r on the vertical axis and Y on the horizontal axis. In the same diagram, graph the interest rate rule (5.2) for given values of π^e, i^*, π^*, Y_n. Call the interest rate rule the monetary policy (MP) relation.

c. Using the diagram you drew in part (b), show that an increase in government spending leads to an increase in output and the real interest rate in the short run.

d. Now consider a change in the monetary policy rule. Suppose the central bank reduces its target inflation rate, π^*. How does the fall in π^* affect the MP relation? What happens to output and the real interest rate in the short run?

e. Suppose the economy starts with $Y = Y_n$ and $\pi^e = \pi^*$. Now suppose there is an increase in π^e. Assume that Y_n does not change. Using the diagram you drew in part (b), show how the increase in π^e affects the MP relation. What happens to output and the real interest rate in the short run?

f. Without attempting to model the dynamics of inflation explicitly, assume that inflation and expected inflation will increase over time if $Y > Y_n$, and that they will decrease over time if $Y < Y_n$. Given the effect on output you found in part (e), will π^e tend to return to the target rate of inflation, π^*, over time?

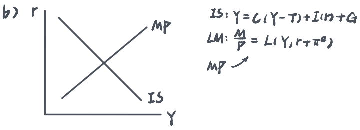
名义利率<->实际利率

a) $i = r + \pi^e$ 代入 (5.1)式

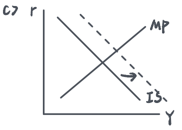
$r + \pi^e = r^* + \pi^* + a(\pi^e - \pi^*) + b(Y - Y_n)$

$r = r^* + (a-1)(\pi^e - \pi^*) + b(Y - Y_n)$

b)

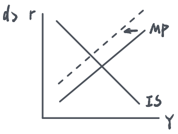


c)



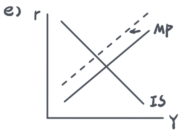
IS↑, Y↑, r↑

d)



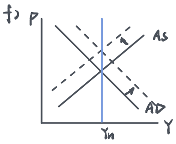
π^*↓, MP↑, Y↓, r↑

e)



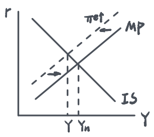
π^e↑, MP↑, Y↓, r↑

f)



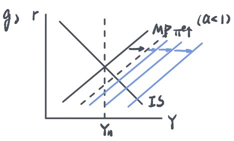
$Y > Y_n$, π^e↑

$Y < Y_n$, π^e↓



$Y > Y_n$同理

g)



π^e↑, MP↓, Y↑, r↓

h) $Y > Y_n$后, π^e进一步上升, 不稳定系统

44 (PS5.7). c. MP does not shift. IS shifts rightward. d. $\pi^*\downarrow$ shifts MP upward. In the short run, $Y\downarrow$ and $r\uparrow$. e. $\pi^e\uparrow$ shifts MP upward. In the short run, $Y\downarrow$ and $r\uparrow$. f. Yes, π^e will return to π^* over time. g. $\pi^e\uparrow$ shifts MP downward. $Y\uparrow, r\downarrow$ in the short run. h. No, π^e will not return to π^*. $a < 1$ is not sensible.

g. Redo part (e), but assuming this time that $a < 1$. How does the increase in π^e affect the MP relation when $a < 1$? What happens to output and the real interest rate in the short run?

h. Again assume that inflation and expected inflation will increase over time if $Y > Y_n$, and that they will decrease over time if $Y < Y_n$. Given the effect on output you found in part (g), will π^e tend to return to the target rate of inflation, π^*, over time? Is it sensible for the parameter a (in the interest rate rule) to have values less than 1?

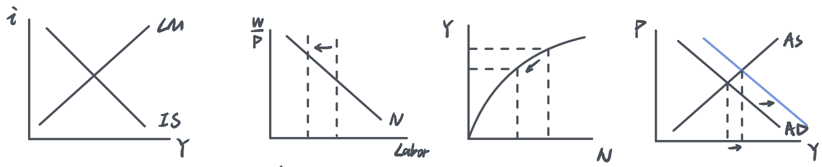
5.8. In a closed AS–AD model, the IS relation is given by $Y = C(Y - T) + I(i - \pi^e) + G$, where T is the lump-sum tax, i is the nominal interest rate, and π^e is the expected rate of inflation. The LM relation is given by $M/P = L(Y, i)$. The production function is given by $Y = F(K, N)$, where K is the constant capital and N is labor input. Suppose F satisfies $F_N' > 0$ and $F_{NN}'' < 0$. Profit maximization gives the demand for labor: $W/P = F_N'(K, N)$, where W is the nominal wage rate. If W is rigid in the short run, find the short-run effect of an increase in G on the equilibrium level of Y. (Calculate partial derivatives)

a) $Y = C(Y-T) + I(i-\pi^e) + G$

$\frac{M}{P} = L(Y, i)$

生产函数: $Y = F(K, N)$

利润最大: $\frac{W}{P} = F_N'(K, N)$



Sticky Wage. ⇒ AS

$$\begin{cases} Y - C(Y-T) - I(i-\pi^e) - G = 0 \\ \frac{M}{P} - L(Y, i) = 0 \\ \frac{W}{P} - F_N(K, N) = 0 \end{cases} \qquad (Y(N))$$

$$-I' di + 0 dP + (1-C_Y) F_N \cdot dN - \overset{(外生)}{dG} = 0$$
$$-L_i di - \frac{M}{P^2} dP - L_Y F_N dN = 0$$
$$0 di - \frac{W}{P^2} dP - F_{NN} dN = 0$$

$$\begin{pmatrix} -I' & 0 & (1-C_Y)F_N \\ -L_i & -\frac{M}{P^2} & -L_Y F_N \\ 0 & -\frac{W}{P^2} & -F_{NN} \end{pmatrix} \begin{pmatrix} di \\ dP \\ dN \end{pmatrix} = \begin{pmatrix} dG \\ 0 \\ 0 \end{pmatrix}$$

$$dN = \frac{1}{\Delta} \begin{vmatrix} -I' & 0 & dG \\ -L_i & -\frac{M}{P^2} & 0 \\ 0 & -\frac{W}{P^2} & 0 \end{vmatrix} = \frac{1}{\Delta} L_i \frac{W}{P^2} dG$$

$$\Delta = -I' \frac{M}{P^2} F_{NN} + L_i \frac{W}{P^2} (1-C_Y) F_N + I' \frac{W}{P^2} L_Y F_N < 0$$

故 $\frac{\partial Y}{\partial G} = \underline{F_N} \cdot \underline{\frac{\partial N}{\partial G}} > 0$

葵花宝典

($\overline{X}$: undisturbed X. Y_n: the potential output)

1 (PS1.1). a. GDP=7580; b. GNP=7700, NNP=NI=7400; c. GNFI=6920, NNFI=6620; d. PI=6010, DPI=5010.

2 (PS1.2). a. GDP=\$150 million; b. CA=–\$10 m; c. GNP=\$150 m; d. GNP = GDP + NFP = 150–25 = \$125 m.

3 (PS1.3). a. GDP=\$150 million; b. Private disposable income=\$135 m, private sector saving=\$35 m, public sector saving=–\$10 m, national saving=\$25 m, government deficit=\$10 m.

4 (PS1.4). a. Easy; b. Private saving, which is not used to finance domestic investment, is either lent to the domestic government to finance its deficit, or lent to foreigners.

5 (PS1.5). a. $S^G = -10$; b. $G^C = 30$; c. $TR = 15$; d. $I = 25$; e. $NX = -15$; f. $GDP = 120$; g. $GNP = 130$; h. $Y^d = 110$.

6 (PS1.6). a. 23.495828%; b. 36.846728%; c. 100.748599%, -15.815104%.

7 (PS1.7). a. Participation rate=50%. b. Labor force=50 million. c. The number of employed workers=47.5 m. d. Employment/Population ratio = 47.5%.

8 (PS1.8). a. Real; b. $Q_t = \frac{R\beta}{1-\beta}$, $\beta = \frac{1}{1+r+\bar{\alpha}}$; c. ↑; d. ↑.

9 (PS2.1). a. MPL↑; b. MPH↓; c. 1/3, 1/3, 2/3; d. The ratio = $1+L/H$; e. More egalitarian.

10 (PS2.2). a. Move rightward; unchanged. b. Supply is unchanged; demand shifts rightward; $r \uparrow$. c. Unchanged; rise; fall.

11 (PS2.3). Easy.

12 (PS2.4). Small Home: a. $\overline{r}$, ϵ depreciating, $NX\uparrow$, $\overline{Y}$, $C\downarrow$. b. $\overline{r}$, ϵ depreciating, $\overline{NX}$, $\overline{Y}$, $\overline{C}$. c. $r\uparrow$, $I\downarrow$, ϵ depreciating, $NX\uparrow$. Large Home: a. $r\downarrow$, ϵ depreciating, $NX\uparrow$, $\overline{Y}$, $C\downarrow$. b. $\overline{r}$, $\overline{NFI}$, $\overline{NX}$, ϵ depreciating, $\overline{Y}$, $\overline{C}$. c. $r\uparrow$, $I\downarrow$, ϵ depreciating, $NX\uparrow$.

13 (PS2.5). NX and r^* are undisturbed.

14 (PS2.6). $\frac{\partial Y}{\partial A} = F_A$, $\frac{\partial r}{\partial A} = \frac{1-C'}{I'}F_A$, $\frac{\partial P}{\partial A} = -\frac{P}{Y}F_A$.

15 (PS2.7). Perfect capital mobility: $\frac{\partial Y}{\partial A} = F_A$, $\frac{\partial r}{\partial A} = 0$, $\frac{\partial \epsilon}{\partial A} = \frac{1-C'}{NX'}F_A$, $\frac{\partial S}{\partial A} = (1-C')F_A$, $\frac{\partial I}{\partial A} = 0$.
Imperfect capital mobility: Let CF denote net capital outflow. $\frac{\partial Y}{\partial A} = F_A$, $\frac{\partial r}{\partial A} = \frac{1-C'}{CF'+I'}F_A$, $\frac{\partial \epsilon}{\partial A} = \frac{1-C'}{NX'}\frac{CF'}{CF'+I'}F_A$, $\frac{\partial S}{\partial A} = (1-C')F_A$, $\frac{\partial I}{\partial A} = I'\frac{1-C'}{CF'+I'}F_A$.

16 (PS3.1). The saving per worker rises. At the time when the saving rate rises, the output per worker does not change. But the output per worker is higher in the new steady state.

17 (PS3.2). a. (i) $\hat{k}^* = 1$; (ii) $\hat{y}^* = 1$; (iii) 0; (iv) 4%; (v) 6%. b. (i) $\hat{k}^* = 0.64$; (ii) $\hat{y}^* = 0.8$; (iii) 0; (iv) 8%; (v) 10%. c. (i) $\hat{k}^* = 0.64$; (ii) $\hat{y}^* = 0.8$; (iii) 0; (iv) 4%; (v) 10%. People in (a) are better off.

18 (PS3.3). a. The break-even line is flatter. b. The break-even line is steeper. c. The actual investment line is lower in $(0,1)$ and higher in $(1,+\infty)$. d. The actual investment line is higher.

19 (PS3.4). a. $s = 0.175$; $MPK = 0.12$; b. $s_{gold} = 0.3, MPK = 0.07$; $K/Y = \frac{0.3}{0.07} \approx 4.2857$.

20 (PS3.5). a. The same. b. Higher in Westland. c. The same. d. Higher in Westland.

21 (PS3.6). $s\hat{y} - (n+g+\delta)\hat{k} = \hat{k}[f'(\hat{k}) - (n+g+\delta)]$.

22 (PS 3.7). It is possible.

23 (PS 3.8). $\frac{\partial K^*}{\partial \xi} = \frac{K^*}{\xi}$. $\frac{\partial Y^*}{\partial \xi} = \frac{Y^*}{\xi}$. $\frac{\partial C^*}{\partial \xi} = \frac{(1-s)Y^*}{\xi}$.

24 (PS3.9). $\gamma_Y = \gamma_K = n + \frac{\alpha}{1-\alpha}g$.

25 (PS 3.10). $\gamma_K = \frac{2-\alpha}{1-\alpha}g + n$. $\gamma_Y = \frac{g}{1-\alpha} + n$.

26 (PS3.11). $\dot{y}/y = sA - (n+\delta)$. If $sA > n+\delta$, y grows indefinitely.

27 (PS 3.12). a. The steady state is determined by $sf(k) = s\tau + (n+\delta)k$. c. $\tau\uparrow\Rightarrow k^*_{High}\downarrow, y^*\downarrow$. The steady-state growth rates of Y^*, C^*, and I^* are n, independent of τ.

28 (PS3.13 ♡). If $\zeta \in [0, 1-c_1)$, $\tau\uparrow\Rightarrow k^*\downarrow, y^*\downarrow$. If $\zeta \in (1-c_1, 1]$, $\tau\uparrow\Rightarrow k^*\uparrow, y^*\uparrow$.

29 (PS 3.14). The rich would like to live in country A. The poor would like to live in A if $s_r/[(s_r - s_p)\chi + s_p] > 1/\alpha_K$.

30 (PS4.1). The saving is undisturbed.

31 (PS4.2). a. $i\downarrow, Y\downarrow$. b. $C_0\downarrow$ leads to $C\downarrow$; $I\downarrow$ if $b_1d_2 - b_2d_1 > 0$, $I\uparrow$ if $b_1d_2 - b_2d_1 < 0$; the private saving changes the same as I. Not necessarily.

32 (PS4.3 ♡). Small China (flexible e): a. $\bar{i}, e \downarrow, \overline{Y}, NX \uparrow$; b. $\bar{i}, e \downarrow, \overline{Y}, \overline{NX}$; c. $\bar{i}, e \downarrow, Y \uparrow, NX \uparrow$. Small China (fixed e): a. $\bar{i}, \bar{e}, Y \downarrow, \overline{NX}(\bar{\epsilon}), M \downarrow$; b. $\bar{i}, \bar{e}, Y \downarrow, NX \downarrow, M \downarrow$; c. $\bar{i}, \bar{e}, \overline{Y}, \overline{NX}, M \downarrow$.

Large China (flexible e): a. $i \downarrow, e \downarrow, Y \downarrow, NX \uparrow, Y^* \downarrow$; b. $\bar{i}, e \downarrow, \overline{Y}, \overline{NX}, \overline{Y}^*$; c. $i \downarrow, e \downarrow, Y \uparrow, NX \uparrow, Y^* \downarrow$. Large China (fixed e): a. $\bar{i}, \bar{e}, Y \downarrow, \overline{NX}, M \downarrow, \overline{Y^*}$; b. $i \uparrow, \bar{e}, Y \downarrow, NX \downarrow, M \downarrow, Y^* \uparrow$; c. $\bar{i}, \bar{e}, \overline{Y}, \overline{NX}, M \downarrow, \overline{Y}^*$.

33 (PS4.4 ♡). a. Fiscal expansion. b. $Y \uparrow, NX \uparrow$.

34 (PS4.5). a. In the short run, i^* rises if anything shifts the world's IS rightward or shifts the world's LM leftward. In the long run, anything that raises the world's demand for investment or lowers the world's saving will raise i^*. b. $Y \uparrow, e \downarrow, NX \uparrow, I \downarrow, C \uparrow$. c. $Y \downarrow, \bar{e}, \overline{NX}, M \downarrow, I \downarrow, C \downarrow$.

35 (PS4.6). Floating e: $e \uparrow$, $Y \downarrow$. Fixed e: $Y \uparrow$.

36 (PS4.7). a. Upward-sloping LM. b. $e \uparrow$, $Y \uparrow$. c. $e \downarrow$, $Y \uparrow$.

37 (PS 4.8). $\frac{\partial Y}{\partial \pi^e} = \frac{1}{\Delta}[L_r(I' + CF')] > 0$. $\frac{\partial r}{\partial \pi^e} = \frac{1}{\Delta}[L_r(1 - C')] < 0$. $\frac{\partial \epsilon}{\partial \pi^e} = \frac{CF'}{NX'}\frac{1}{\Delta}[L_r(1 - C')] < 0$. $\Delta = -L_r(1 - C') - (I' + CF')L_Y > 0$.

38 (PS5.1 ♡). a. $L^D = 25/(\frac{W}{P})^2$. b. $Y_n = 25$. c. $Y = 25P$.

39 (PS5.2). a. In the short run, $Y \downarrow, i \downarrow, P \downarrow, C \downarrow$; for $I(i - \pi^e)$, investment and private saving rise; for $I(i - \pi^e, \underset{+}{Y})$, investment and private saving are ambiguous and the paradox may occur. b. In the long run, $Y = Y_n$. $i \downarrow, P \downarrow, C \downarrow$; investment and private saving rise. The paradox disappears.

40 (PS5.3). See the liquidity trap.

41 (PS5.4 ♡). It is possible.

42 (PS5.5 ♡). a. $(Y, P) = (1000, 6)$. b. $(Y, P) = (1600, 2)$.

43 (PS5.6). a. $\pi^e = 5$, $u = 5$, $L = 6.25$. b. $\pi^e = 0$, $u = 5$, $L = 5$. c. Recommend (b). d. $u = 2.5$, $L = 3.75$. e. Dynamic inconsistency.

44 (PS5.7). c. MP does not shift. IS shifts rightward. d. $\pi^* \downarrow$ shifts MP upward. In the short run, $Y \downarrow$ and $r \uparrow$. e. $\pi^e \uparrow$ shifts MP upward. In the short run, $Y \downarrow$ and $r \uparrow$. f. Yes, π^e will return to π^* over time. g. $\pi^e \uparrow$ shifts MP downward. $Y \uparrow, r \downarrow$ in the short run. h. No, π^e will not return to π^*. $a < 1$ is not sensible.

45 (PS 5.8). $\frac{\partial Y}{\partial G} = \frac{-L_i}{\Delta}F_N > 0$. $\Delta = -(1 - C')F_N L_i + I'(F_{NN}M/W - L_Y F_N) > 0$.