A Toy Model for Government Policy

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Model highlights

- "Negative externalities" - "market failure" - "government intervention" - tax policy (macroprudential policy, MaP)
- A wedge between the private return and the social return of borrowing
- "Over-borrowing" or inefficient borrowing
- Optimal tax policy (macroprudential policy)

The model

- Reference: Jeanne, O., 2014, "Macroprudential Policies in A Global Perspective", NBER working paper 19967.
- Two time periods $t \in \{1, 2\}$
- Two types of agents, one unit mass of **Borrowers** (b) and one unit mass of **Lenders** (l)
- Period 1: pre-crisis period or "**credit boom**" stage
- Period 2: crisis period or "**deleveraging**" stage
- A small open economy model, why?

The model

- Lending and investment take place in the first period
- Repayment takes place (**or not**) in the second place
- The model is real
- One single good used for **consumption** and **investment**
- A mass of identical lenders and a mass of identical borrowers

Lenders

- Lenders receive endowments Y in the first period
- Utility function:

$$U_l = u(C_l) + E(C_l^{'})$$

  where $C^{'}$ denotes consumption in the second period
- Lenders lend their savings, $S = Y - C_l$, at the riskless interest rate, r

Lenders

- Lenders' consumption - saving problem: to determine C_l or S
- Usually, the problem depends on interest rates, risk preference, etc.
- Formally, maximize U_l subject to budget constraint $C_l = Y - S$
- The Lagrange:

$$L = \max_{C_l, S}[u(C_l) + (1+r)S] + \lambda(Y - S - C_l)$$

$$\begin{aligned} \partial L / \partial C_l &= 0 \\ \partial L / \partial S &= 0 \end{aligned}$$

That is:

$$\begin{aligned} u'(C_l) - \lambda &= 0 \\ (1+r) - \lambda &= 0 \end{aligned}$$

Optimal consumption - saving plans

- Optimal consumption - saving plans:

$$u'(C_l) = (1 + r)$$

- Alternatively, lenders save until the marginal benefit of saving (i.e. $1 + r$) is equal to the marginal cost $u'(C_l)$
- If $u(C) = \log C$, then $u'(C_l) = \frac{1}{C_l}$
- Optimal consumption is: $C_l = \frac{1}{1+r}$; optimal saving is: $S = Y - C_l = Y - \frac{1}{1+r}, S'(.) > 0$

Borrowers

- Borrowers are identical atomistic entrepreneurs (or firms) who need funds to finance investment projects
- A given entrepreneur invests I at date 1 in the hope of receiving $f(I)$ of good at date 2, where $f'(I) > 0, f''(I) < 0$
- Moral hazard: the project is risky, with $1 - p$ default rate
- The payoff: $\left\{ {f(I), p \atop 0, 1-p} \right\}$, expected payoff $pf(I)$

Borrowers

- Assume borrowers have no funds at date 1, implying that the investment is entirely financed by debt, $D = I$
- Because of moral hazard, borrowers have to pay a default risk premium, they promise to pay $(1+r)\frac{D}{p}$
- Expected debt payments: $(1+r)D = p(1+r)\frac{D}{p}$

Borrowers

- The borrowers consume in the second period only (e.g. because the agency cost of debt deters them from borrowing to finance consumption)
- Utility function:
$$U_b = E(C_b^{'})$$
- Domestic welfare $U = U_l + U_b = u(C_l) + E(C_l^{'}) + E(C_b^{'})$

Systemic risk

- Assume the default rate is an increasing function of the **aggregate** level of debt

$$p = p(D), p'(.) < 0$$

- Intuition: higher aggregate debt – "bad state" – higher probability of default – bad for everyone (when default, they get nothing)
- Negative externality: individuals do not take into account the impact of their borrowing on the risk of default for the other borrowers
- Over-borrowing – negative externalities – market failure – government intervention (MaP policy)

Decentralized equilibrium (DE)

- Borrowers' ex-ante utility under laissez-faire (lf):

$$U_b = pf(I) - (1+r)I$$

- Borrowers' problem: how much to borrow (or invest in the first period)
- The entrepreneur borrows until the marginal benefit is equal to the marginal cost of borrowing, $pf'(I) = (1+r)$, we get I_{lf}

Centralized equilibrium (CE)

- Assume there exists a social planner (sp) who internalizes this externality, i.e. $p(D)$ or $p(I)$
- The social planner maximizes:

$$p(I)f(I) - (1+r)I$$

- Borrowers' problem: how much to borrow (or invest in the first period)
- The entrepreneur borrows until the marginal benefit is equal to the marginal cost of borrowing, $pf'(I) + p'f(I) = (1+r)$, we get I_{sp}

Over-borrowing

- Since $p' < 0, f(I) > 0, p'f(I) < 0$, and $pf'(I) + p'f(I) < pf'(I)$
- $I_{sp} < I_{lf}$
- Intuition: the social planner internalizes this negative externality induced by over-borrowing, he chooses a lower level of debt

DE and CE

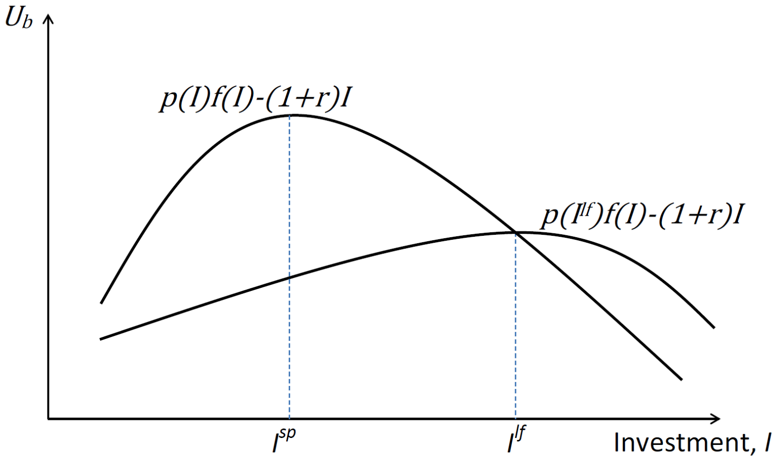

Figure 1: Borrowers' welfare under laissez-faire and a social planner

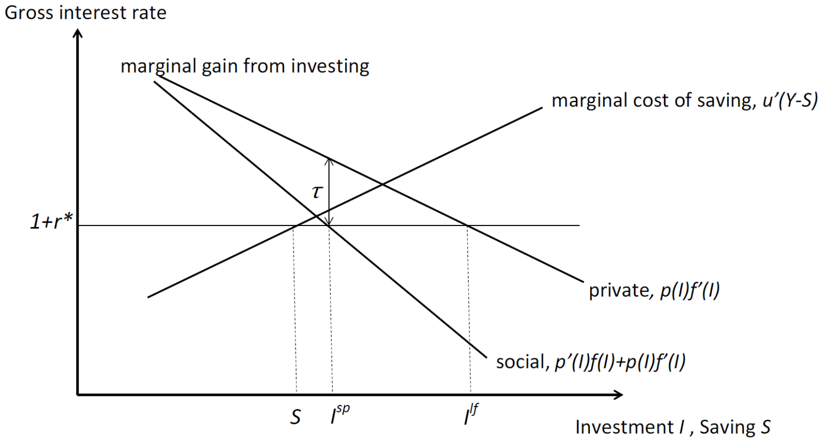

Figure 2: Metzler diagram with systemic debt externalities

Optimal macroprudential policy

- A tax on borrowing to remove the wedge between the private return and the social return
- Higher taxes – higher costs of borrowing – lower amount of borrowing
- The social planner maximizes:

$$pf'(I) = 1 + r + \tau^*$$

$$\tau^* = -p'(I_{sp})f(I_{sp})$$

- Optimal MaP tax is increasing with the level of debt
- This is countercyclical MaP policy

Additional questions

- $p(I)$ or p?
- Lenders, why do we model?
- Is moral hazard important in our model? Why do we need p?
- Borrowers' utility:

$$
\begin{aligned}
U_b &= [pf(I) + (1-p) * 0] - [p\frac{(1+r)}{p}I + (1-p) * 0] \\
&= pf(I) - (1+r)I
\end{aligned}
$$

Additional questions

- DE vs. CE; Laissez-Faire (LF) vs. Social Planner (SP)
- Marginal cost of borrowing: $(1+r)$ or $\frac{(1+r)}{p}$?
- $I \uparrow \Rightarrow p \downarrow \Longrightarrow$ lower welfare for borrowers;
  Large borrowing (or investment) ex-ante $\Longrightarrow$ Higher probability of default, lower welfare ex-post
- $p'f(I)$, intuition?
- Figure 1
- Does the debt crisis affect lenders?