The JK Model for Government Policy

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Background

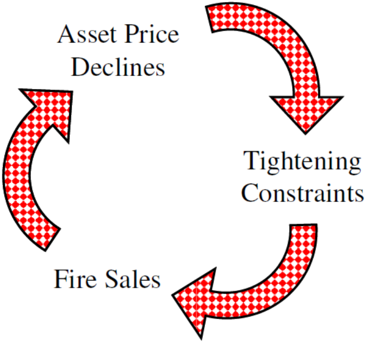
- Financial crisis by large capital flows (most relevant to EMEs)
- To understand the "boom" - "burst" cycle
- The "boom" period: large capital flows $\Rightarrow$ large volatility $\Rightarrow$ more vulnerable (fragile) $\Rightarrow$ higher systemic risks
- The "burst" period: deleveraging stage $\Rightarrow$ lower aggregate demand $\Rightarrow$ lower asset prices $\Rightarrow$ financial crisis

Tax policy (Macroprudential policy)

- Large capital flows can be inefficient; "over-borrowing"
- Capital controls as a MaP
- A borrowing constraint that depends on asset prices can be used to engineer the crisis

Why the financial crisis is so devastating?

- The key is a negative feedback loop in the crisis period



How do we model the crisis?

- A small open economy model
- Domestic borrowers and international lenders
- **A borrowing constraint that depends on asset prices**
- Financial amplification and "boom" - "burst" cycle
- Agents do not internalize the effects of their borrowing on asset prices
- "Over-borrowing" - "negative externalities"

The model

- Reference: Jeanne, O. and A. Korinek, 2010, "Excessive Volatility in Capital Flows: A Pigouvian Taxation Approach", *American Economic Association: Papers and Proceedings*.
- Three time periods $t \in \{0, 1, 2\}$
- One type of agents, one unit mass of **Borrowers**
- International lenders
- Period 0: pre-crisis period or "**boom**" stage
- Period 1: crisis period or "**burst**" stage
- Period 2: the long term

The model

- A small open economy model
- Lending and borrowing take place at date 0 (d_1) and date 1 (d_2)
- Repayment takes place at date 1 and date 2
- The model is real
- One single good used for consumption
- The riskless world interest rate is normalized to zero

Consumers

- The utility of the representative consumer is:

$$U = u(c_0) + u(c_1) + c_2$$

- The first-best level of consumption is:
  $u'(c_0^*) = u'(c_1^*), i.e. c_0^* = c_1^* = c^*$

Domestic income

- Agents receive one unit of asset at date 0
- A stochastic endowment e at date 1
- Return y on the asset that materializes at date 2

Budget constraint

- The budget constraints of a domestic consumer are:

$$\begin{aligned} c_0 &= d_1 + (1-\theta_1)p_0 \\ c_1 + d_1 &= e + d_2 + (\theta_1 - \theta_2)p_1 \\ c_2 + d_2 &= \theta_2 y \end{aligned}$$

Assumptions on the asset

- The asset can be traded in a competitive market
- The asset can only be exchanged in the domestic market, international lenders cannot buy the asset
- In a symmetric equilibrium, $\theta_t = 1$, other equilibria?

Budget constraint

- The budget constraints of a domestic consumer are simplified:

$$\begin{aligned} c_0 &= d_1 \\ c_1 + d_1 &= e + d_2 \\ c_2 + d_2 &= y \end{aligned}$$

The borrowing constraint

- Each consumer faces a collateral constraint:

$$d_2 \leq \theta_1 p_1$$

- The micro-foundation is that a consumer could walk away and lenders can seize his asset and sell it to other consumers

Period 1 equilibrium

- Utility is maximized subject to budget constraints and the borrowing constraint:

$$V_{lf} = \max_{d_2,\theta_2} u(c_1) + c_2 + \lambda_{lf}(\theta_1 p_1 - d_2)$$

- Simplified problem:

$$\max_{d_2,\theta_2} u(e + d_2 + (\theta_1 - \theta_2)p_1 - d_1) + \theta_2 y - d_2 + \lambda_{lf}(\theta_1 p_1 - d_2)$$

- $\lambda_{lf} \geq 0$ is the shadow cost of the constraint
- Where is $u(c_0)$?

First-order conditions (FOCs)

- First-order condition for θ_2 :

$$u'(c_1)(-p_1) + y = 0$$

i.e.

$$p_1 = \frac{y}{u'(c_1)}$$

- First-order condition for d_2 :

$$u'(c_1) - 1 - \lambda_{lf} = 0$$

i.e.

$$u'(c_1) = 1 + \lambda_{lf}$$

Net worth

- Define $m_1 = e - d_1$ as the liquid net worth
- m_1 is the only **state variable**
- If $u(c) = \log c$, then $p_1 = \frac{y}{u'(c_1)} = yc_1$
- This is meant to capture asset prices are usually positively correlated to aggregate consumption or state

Constrained equilibrium

- When the borrowing constraint binds, i.e. $\lambda_{lf} > 0$ and $d_2 = \theta_1 p_1 = p_1$
- Then, $c_1 = m_1 + d_2 = m_1 + p_1 = m_1 + \frac{y}{u'(c_1)}$
- The above equation determines $c_1(m_1)$
- If $u(c) = \log c$, then $c_1 = \frac{m_1}{1-y} = \frac{m_1}{m^*}$, where $m^* = 1 - y$; $p_1 = yc_1 = y\frac{m_1}{m^*}$

Unconstrained equilibrium

- When the borrowing constraint does not bind, i.e. $\lambda_{lf} = 0$ and $d_2 < p_1$
- $u'(c_1) = 1, c_1 = c^* = 1$
- $p_1 = yc_1 = y$
- The asset price does not depend on the state variable m_1
- $c_0 = 1, d_1 = 1$

When does the constraint bind?

- Conceptually, when the liquid net worth is high enough, the constraint will never bind
- $m_1 \geq m^*$
- Why not p_1?
- Formally, the equilibrium is unconstrained iff the payoff of the asset y is high enough to cover d_2

$$d_2 \leq y$$

That is,

$$d_2 = c^* - e_1 + d_1 = 1 - m_1 \leq y$$

$$m_1 \geq 1 - y \equiv m^*$$

The stochastic variable e

- Since $m_1 \geq m^*$, we have $e \geq m^* + d_1$
- Low realizations of e may trigger a financial crisis
- Financial crisis is defined as an event when the constraint binds with a positive probability, i.e. $m_1 \leq m^*$
- We assume e is uniformly distributed in $[\bar{e} - \varepsilon, \bar{e} + \varepsilon]$

Period 0 equilibrium

- We need to solve for d_1

$$\max_{d_1} u(c_0) + E_0[u(c_1) + c_2]$$

- First-order condition:

$$u'(c_0) - E_0[u'(c_1)] = 0$$

That is:

$$u'(c_0) = E_0[u'(c_1)]$$

- Intuitions

Optimal debt under LF

$$
\begin{aligned}
\frac{1}{d_1} &= E_0[u'(c_1)] \\
&= \int_{\bar{e}-\varepsilon}^{m^*+d_1} \frac{1}{2\varepsilon}\frac{m^*}{e-d_1}de + \int_{m^*+d_1}^{\bar{e}+\varepsilon} \frac{1}{2\varepsilon}de \\
&= \frac{1}{2\varepsilon}[m^*(\log(m^*) - \log(\bar{e}-\varepsilon-d_1))] + \frac{1}{2\varepsilon}(\bar{e}+\varepsilon-m^*-d_1) \\
&= \frac{1}{2\varepsilon}[m^* \log(\frac{m^*}{\bar{e}-\varepsilon-d_1}) + \bar{e}+\varepsilon-m^*-d_1]
\end{aligned}
$$

- This equation determines the level of debt under laissez-faire, d_1^{lf}
- The fixed-point equation can be solved numerically given $m^*, \bar{e}, \varepsilon$

Summary of DE

- When $e \in [m^* + d_1, \bar{e} + \varepsilon]$, the borrowing constraint will never bind
- $c_1 = c^* = 1, p_1 = yc_1 = y, c_0 = 1, d_1 = 1, d_2 = m_1 - 1 = 2 - e$

Summary of DE

- When $e \in \left[\bar{e} - \varepsilon, m^* + d_1\right]$, the borrowing constraint binds with a positive probability
- $c_1 = \frac{m_1}{m^*} < 1, p_1 = yc_1 = \frac{m_1}{m^*}y, d_2 = p_1 = \frac{m_1}{m^*}y$
- d_1 and c_0 need to be solved numerically given $m^*, \bar{e}, \varepsilon$

Centralized equilibrium

- Assume there is a social planner who understands $p_1(m_1)$
- The key is to characterize "over-borrowing"
- We need to prove $d_1^{lf} > d_1^{sp}$

Set up the problem

- Assume the social planner faces the same constraints as private agents, but internalizes $p_1(m_1)$
- Intuitively, SP understands the negative feedback loop
- Would the SP choose a lower level of debt *ex-ante*?

Period 1 equilibrium

- The problem:

$$V_{sp} = \max_{d_2}\{u(c_1) + c_2 + \lambda_{sp}[p(m_1) - d_2]\}$$

where $p(m_1) = y/u'(c_1)$

- First-order condition:

$$u'(c_1) = 1 + \lambda_{sp}$$

- SP chooses the same allocation as under LF in period, i.e., same *ex-post*

Marginal value of net worth

- Marginal value of net worth under CE:

$$V'_{sp} = u'(c_1) + \lambda_{sp} p'(m_1)$$

- Marginal value of net worth under DE:

$$V'_{lf} = u'(c_1)$$

- Since $\lambda_{sp} > 0$, $p'(m_1) > 0$, the marginal value of liquid net worth is higher under CE
- SP wants to mitigate the negative impact of financial crisis by having more net worth to support asset prices
- Less debt *ex-ante*?
- How about consumption?

Period 0 equilibrium

- The Euler equation under CE:

$$u'(c_0) = E_0[u'(c_1) + \lambda_{sp} p'(m_1)]$$

- The Euler equation under DE:

$$u'(c_0) = E_0[u'(c_1)]$$

- Since $\lambda_{sp} > 0$, $p'(m_1) > 0$, $u'(c_0)$ is decreasing, $c_0^{sp} < c_0^{lf}$, $d_1^{sp} < d_1^{lf}$
- Over-borrowing at the social level

Optimal debt under DE

- $\lambda_{sp} = \frac{1}{c_1} - 1 = (\frac{m^*}{m_1} - 1)$
- $p'(m_1) = \frac{1}{m^*} - 1 > 0$, if $m_1 < m^*$

Optimal debt under DE

$$
\begin{aligned}
\frac{1}{d_1} &= E_0[u'(c_1) + \lambda_{sp} p'(m_1)] \\
&= \frac{1}{2\varepsilon} \int_{\bar{e}-\varepsilon}^{m^*+d_1} [\frac{m^*}{m_1} + (\frac{m^*}{m_1} - 1)(\frac{1}{m_*} - 1)] de + \frac{1}{2\varepsilon} \int_{m^*+d_1}^{\bar{e}+\varepsilon} de \\
&= \frac{1}{2\varepsilon} \int_{\bar{e}-\varepsilon}^{m^*+d_1} (\frac{1}{e - d_1} + 1 - \frac{1}{m^*}) de + \frac{1}{2\varepsilon} \int_{m^*+d_1}^{\bar{e}+\varepsilon} de \\
&= 1 + \frac{1}{2\varepsilon} [\log(\frac{m^*}{\bar{e} - \varepsilon - d_1}) - 1 - \frac{d_1 - \bar{e} + \varepsilon}{m^*}]
\end{aligned}
$$

- This equation determines the level of debt under DE, d_1^{sp}

Optimal tax on capital inflows

- Optimal tax policy (macroprudential policy): restoring the SP debt (or consumption) level by having a tax on borrowing

$$u'(c_0) = (1+\tau)E_0[u'(c_1)]$$

- Formally:

$$1+\tau = \frac{E_0[u'(c_1) + \lambda_{sp} p'(m_1)]}{E_0[u'(c_1)]}$$

- That is:

$$\tau = \frac{E_0[\lambda_{sp} p'(m_1)]}{E_0[u'(c_1)]}$$

Optimal Tax policy (MaP policy)

- Also:

$$1 + \tau = \frac{E_0[u'(c_1) + \lambda_{sp} p'(m_1)]}{E_0[u'(c_1)]} = \frac{1}{d_1^{sp} E_0[u'(c_1)]}$$

- And:

$$\begin{aligned} E_0[u'(c_1)] &= \frac{1}{2\varepsilon}[m^* \log(\frac{m^*}{\bar{e} - \varepsilon - d_1^{sp}}) + \bar{e} + \varepsilon - m^* - d_1^{sp}] \\ &= m^*(\frac{1}{d_1^{sp}} - 1) + 1 \end{aligned}$$

- So that:

$$1 + \tau = \frac{1}{m^* + (1 - m^*) d_1^{sp}}$$

Quantitative analysis

- Set parameter values: $\bar{e} = 1.3, m^* = 0.2$
- When the constraint does not bind, i.e. $d_2 < y \Rightarrow 2 - e < y \Rightarrow e > 1 + m^*$
- That is, $\varepsilon < \bar{e} - m^* - 1 = 0.1$

Probability of financial crisis

- Probability of financial crisis:

$$\begin{aligned}\pi &= \frac{1}{2\varepsilon}\int_{\bar{e}-\varepsilon}^{m^*+d_1} de \\ &= \frac{1}{2} - \frac{\bar{e}-m^*-d_1}{2\varepsilon}\end{aligned}$$

- Probability of financial crisis under DE:

$$\pi^{lf} = \frac{1}{2} - \frac{\bar{e}-m^*-d_1^{lf}}{2\varepsilon}$$

- Probability of financial crisis under CE:

$$\pi^{sp} = \frac{1}{2} - \frac{\bar{e}-m^*-d_1^{sp}}{2\varepsilon}$$

Magnitude of financial crisis

- The minimum level of c_1 is:

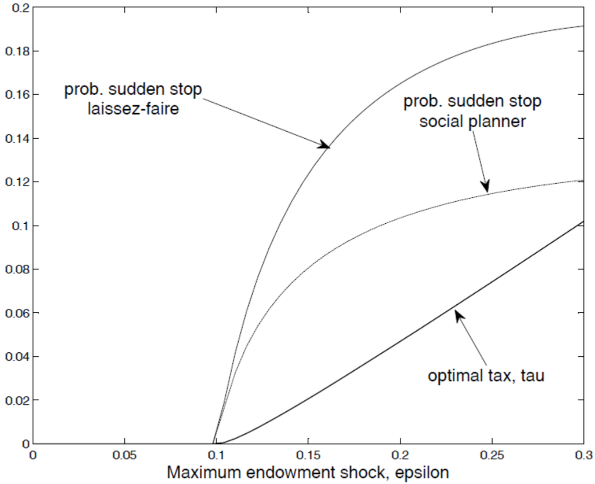
$$\begin{aligned}\min c_1 &= \frac{\min e - d_1}{m^*} = \frac{\bar{e} - \varepsilon - d_1}{m^*} \\ &= 1 - 2\frac{\varepsilon\pi}{m^*}\end{aligned}$$

- The magnitude of financial crisis Δc is the expected consumption gap $c^* - c_1$ conditional on a sudden stop:

$$\begin{aligned}\Delta c &= c^* - c_1 \\ &= 1 - \min c_1 \\ &= 2\frac{\varepsilon\pi}{m^*}\end{aligned}$$

- Δc^{lf} and Δc^{sp}

Probability of financial crisis and optimal tax



Probability of financial crisis and optimal MaP

- The probability of financial crisis reaches around 19% for $\varepsilon = 0.3$ under DE; around 12% under CE
- E.g., $\varepsilon = 0.13, \pi^{lf} = 0.1, \pi^{sp} = 0.068, \tau = 0.013$
- E.g., $\varepsilon = 0.3, \pi^{lf} = 0.19, \pi^{sp} = 0.12, \tau = 0.1$

Summary

- The probability of financial crisis is reduced significantly under CE
- The magnitude of financial crisis is reduced significantly under CE